

Towards the supersymmetric standard model from intersecting D6-branes on the \mathbb{Z}'_6 orientifold

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Abstract

We construct $\mathcal{N} = 1$ supersymmetric fractional branes on the \mathbb{Z}'_6 orientifold. Intersecting stacks of such branes are needed to build a supersymmetric standard model. If a, b are the stacks that generate the $SU(3)_c$ and $SU(2)_L$ gauge particles, then, in order to obtain *just* the chiral spectrum of the (supersymmetric) standard model (with non-zero Yukawa couplings to the Higgs multiplets), it is necessary that the number of intersections $a \circ b$ of the stacks a and b , and the number of intersections $a \circ b'$ of a with the orientifold image b' of b satisfy $(a \circ b, a \circ b') = \pm(2, 1)$ or $\pm(1, 2)$. It is also necessary that there is no matter in symmetric representations of the gauge group, and not too much matter in antisymmetric representations, on either stack. We provide a number of examples having these properties. Different lattices give different solutions and different physics.

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1 Introduction

One of the main phenomenological attractions of using D-branes is that they permit a “bottom-up” approach to constructing the standard model from Type II string theory. Open strings that begin and end on a stack a of N_a D-branes generate the gauge bosons of a (supersymmetric) $U(N_a)$ gauge theory living in the world volume of the D-branes. In the original bottom-up models [1, 2, 3, 4] a stack of D3-branes is placed at an orbifold T^6/\mathbb{Z}_N singularity and the standard model gauge group (possibly augmented by additional $U(1)$ factors) is obtained by choosing a suitable embedding γ_θ of the action of the generator θ of the orbifold point group \mathbb{Z}_N on the Chan-Paton indices of the D3-branes. Besides the gauge bosons, fermionic matter also survives the orbifold projection. So long as only D3-branes are retained, the fermion spectrum generally makes the non-abelian gauge symmetries anomalous, reflecting the fact that a general collection of D3-branes has uncanceled Ramond-Ramond (RR) tadpoles. The required cancellation is achieved by introducing D7-branes, which generate further gauge symmetries, and additional fermions. When all tadpoles are cancelled, so are the gauge anomalies. However, we showed in an earlier paper [5] that all such models, whether utilising fixed points on an orbifold or an orientifold, have electroweak Higgs content that is non-minimal, both for the (non-supersymmetric) Standard Model or its supersymmetric extension, the MSSM. As a consequence there is a generic flavour changing neutral current (FCNC) problem in such models, and we conclude that such models are not realistic. (See, however, [6], which argues that a supersymmetric, standard-like model with three Higgs doublets, derived from compactifying the $E_8 \otimes E_8$ heterotic string on a \mathbb{Z}_3 orbifold, *can* circumvent the FCNC problem without an excessively heavy Higgs sector.)

An alternative approach that also uses D-branes is “intersecting brane” model building [7]. In these models one starts with two stacks, a and b with $N_a = 3$ and $N_b = 2$, of D4-, D5- or D6-branes wrapping the three large spatial dimensions plus respectively 1-, 2- and 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold) on which the theory is compactified. These generate the gauge group $U(3) \times U(2) \ni SU(3)_c \times SU(2)_L$, and the non-abelian component of the standard model gauge group is immediately assured. Further, (four-dimensional) fermions in bifundamental representations $(\mathbf{N}_a, \overline{\mathbf{N}}_b) = (\mathbf{3}, \overline{\mathbf{2}})$ of the gauge group can arise at the multiple intersections of the two stacks. These are precisely the representations needed for the quark doublets Q_L of the Standard Model. For D4- and D5-branes, to get *chiral* fermions the stacks must be at a singular point of the transverse space. In general, intersecting branes yield a non-supersymmetric spectrum, so that, to avoid the hierarchy problem, the string scale associated with such models must be low, no more than a few TeV. Then, the high energy (Planck) scale associated with gravitation does not emerge naturally. Nevertheless, it seems that these problems can be surmounted [8, 9], and indeed an attractive model having just the spectrum of the Standard Model has been constructed [10]. It uses D6-branes that wrap 3-cycles of an orientifold T^6/Ω , where Ω is the world-sheet parity operator. The advantage and, indeed, the necessity of using an orientifold stems from the fact that for every stack a, b, \dots there is an orientifold image a', b', \dots . At intersections of a and b there are chiral fermions in the $(\mathbf{3}, \overline{\mathbf{2}})$ representation of $U(3) \times U(2)$, where the $\mathbf{3}$ has charge $Q_a = +1$ with respect to the $U(1)_a$ in $U(3) = SU(3)_c \times U(1)_a$, and the $\overline{\mathbf{2}}$ has charge $Q_b = -1$ with respect to the $U(1)_b$ in $U(2) = SU(2)_L \times U(1)_b$. However, at intersections of a and b' there are chiral fermions in the $(\mathbf{3}, \mathbf{2})$ representation, where the $\mathbf{2}$ has $U(1)_b$ charge $Q_b = +1$. In the model of [10], the number of intersections $a \circ b$ of the stack a with b is 2, and the number of intersections $a \circ b'$ of the stack a with b' is 1. Thus, as required for the Standard Model, there are 3 quark doublets. These have net $U(1)_a$ charge $Q_a = 6$, and net $U(1)_b$ charge $Q_b = -3$. Tadpole cancellation requires that overall both charges, sum to zero, so further fermions are essential, and indeed required by the Standard Model. 6 quark-singlet states u_L^c and d_L^c belonging to the $(\mathbf{1}, \overline{\mathbf{3}})$ representation of $U(1) \times U(3)$, having a total of $Q_a = -6$ are sufficient to ensure overall cancellation of Q_a , and these arise from the intersections of a with other stacks c, d, \dots having just a single D6-brane. Similarly, 3 lepton doublets L , belonging to the $(\mathbf{2}, \overline{\mathbf{1}})$ representation of $U(2) \times U(1)$, having a total $U(1)_b$ charge of $Q_b = 3$, are sufficient to ensure overall cancellation of Q_b , and these arise from the intersections of b with other stacks having just a single D6-brane. In contrast, had we not used an orientifold, the requirement of 3 quark doublets would necessitate having the number of intersections $a \circ b = 3$. This makes no difference to the charge $Q_a = 6$ carried by the quark doublets, but instead the $U(1)_b$ charge carried by the quark doublets is $Q_b = -9$,

which cannot be cancelled by just 3 lepton doublets L . Consequently, additional vector-like fermions are unavoidable unless the orientifold projection is available. This is why the orientifold is essential if we are to get just the matter content of the Standard Model or of the MSSM.

Actually, an orientifold can allow the standard-model spectrum without vector-like matter even when $a \circ b = 3$ and $a \circ b' = 0$ [11]. This is because in orientifold models it is also possible to get chiral matter in the symmetric and/or antisymmetric representation of the relevant gauge group from open strings stretched between a stack and its orientifold image. Both representations have charge $Q = 2$ with respect to the relevant $U(1)$. The antisymmetric (singlet) representation of $U(2)$ can describe a lepton single state ℓ_L^c , and 3 copies contribute $Q_b = 6$ units of $U(1)_b$ charge. If there are also 3 lepton doublets L belonging to the bifundamental representation $(\mathbf{2}, \mathbf{\bar{1}})$ representation of $U(2) \times U(1)$, each contributing $Q_b = 1$ as above, then the total contribution is $Q_b = 9$ which **can** be cancelled by 3 quark doublets Q_L in the $(\mathbf{3}, \mathbf{\bar{2}})$ representation of $U(3) \times U(2)$. Thus, as asserted, orientifold models can allow just the standard-model spectrum even when $(a \circ b, a \circ b') = (3, 0)$.

Despite the attractiveness of the model in [10], there remain serious problems in the absence of supersymmetry. A generic feature of intersecting brane models is that flavour changing neutral currents are generated by four-fermion operators induced by string instantons [12]. The severe experimental limits on these processes require that the string scale is rather high, of order 10^4 TeV. This makes the fine tuning problem very severe, and the viability of such models highly questionable. Further, in non-supersymmetric theories, such as these, the cancellation of RR tadpoles does not ensure Neveu Schwarz-Neveu Schwarz (NSNS) tadpole cancellation. NSNS tadpoles are simply the first derivative of the scalar potential with respect to the scalar fields, specifically the complex structure and Kähler moduli and the dilaton. A non-vanishing derivative of the scalar potential signifies that such scalar fields are not even solutions of the equations of motion. Thus a particular consequence of the non-cancellation is that the complex structure moduli are unstable [13]. One way to stabilise these moduli is for the D-branes to wrap an orbifold T^6/P rather than a torus T^6 . The FCNC problem can be solved and the complex structure moduli stabilised when the theory is supersymmetric. First, a supersymmetric theory is not obliged to have the low string scale that led to problematic FCNCs induced by string instantons. Second, in a supersymmetric theory, RR tadpole cancellation ensures cancellation of the NSNS tadpoles [14, 15]. An orientifold is then constructed by quotienting the orbifold with the world-sheet parity operator Ω . (As explained above, an orientifold is necessary to allow the possibility of obtaining just the spectrum of the supersymmetric standard model.)

Several attempts to construct the MSSM using D6-branes and a \mathbb{Z}_4 , $\mathbb{Z}_4 \times \mathbb{Z}_2$ or \mathbb{Z}_6 orientifold have been made [16, 17, 18, 19]. The most successful attempt to date is the last of these [19, 20], which uses D6-branes intersecting on a \mathbb{Z}_6 orientifold to construct an $\mathcal{N} = 1$ supersymmetric standard-like model using 5 stacks of branes. We shall not discuss this beautiful model in any detail except to note that the intersection numbers for the stacks a , which generates the $SU(3)_c$ group, and b , which generates the $SU(2)_L$, are $(|a \circ b|, |a \circ b'|) = (0, 3)$. In this case it is impossible to obtain lepton singlet states ℓ_L^c as antisymmetric representations of $U(2)$. Further, it was shown, quite generally, that it is impossible to find stacks a and b such that $(|a \circ b|, |a \circ b'|) = (2, 1)$ or $(1, 2)$. Thus, as explained above, it is impossible to obtain exactly the (supersymmetric) Standard Model spectrum.

The question then arises as to whether the use of a different orientifold could circumvent this problem. In this paper we address this question for the \mathbb{Z}'_6 orientifold. We do not attempt to construct a standard(-like) MSSM. Instead, we merely see whether there are any stacks a, b that simultaneously satisfy the supersymmetry constraints, the absence of chiral matter in symmetric representations of the gauge groups (see below), which have not too much chiral matter in antisymmetric representations of the gauge groups (see below), and which have $(|a \circ b|, |a \circ b'|) = (2, 1)$ or $(1, 2)$. We do not pursue the alternative that $(|a \circ b|, |a \circ b'|) = (3, 0)$ or $(0, 3)$, since such models, with 3 lepton singlet states arising on the $U(2)$ stack, do not have the standard-model couplings of these states to the Higgs multiplet. With $(N_a, N_b) = (3, 2)$ we explained above why $(a \circ b, a \circ b') = (2, 1)$ is sufficient to ensure that no vector-like matter is necessary to ensure that the net $U(1)_b$ charge Q_b is zero, and the same is obviously the true if $(a \circ b, a \circ b') = (1, 2)$; it amounts to interchanging b and b' . Intersection numbers $(a \circ b, a \circ b') = (-2, -1)$ or $(-1, -2)$ are equally acceptable, since negative intersection numbers correspond to opposite chiralities. Thus $(a \circ b, a \circ b') = (\pm 2, \pm 1)$, where underlining signifies any permutation, is sufficient. For calculational

purposes it is convenient to let *either* stack a or b generate the $SU(3)_c$ gauge group, so that $(N_a, N_b) = (3, 2)$. Interchanging a and b gives $(b \circ a, b \circ a') = (-a \circ b, a \circ b')$. Thus the intersection numbers are generally required to satisfy $(|a \circ b|, |a \circ b'|) = (2, 1)$. If $a \circ b$ and $a \circ b'$ have the same sign, then $N_a = 3$ and $N_b = 2$; otherwise $N_a = 2$ and $N_b = 3$. In what follows we parallel quite closely the treatment [19] of Honecker & Ott for the \mathbb{Z}_6 orientifold.

2 The \mathbb{Z}'_6 orbifold

We assume, as is customary, that the torus T^6 factorises into three 2-tori $T^2_1 \times T^2_2 \times T^2_3$. The three 2-tori T^2_k ($k = 1, 2, 3$) are parametrised by three complex coordinates z_k . The action of the generator θ of the point group \mathbb{Z}'_6 on the coordinates z_k is given by

$$\theta z_k = e^{2\pi i v_k} z_k \quad (1)$$

where

$$(v_1, v_2, v_3) = \frac{1}{6}(1, 2, -3) \text{ for } \mathbb{Z}'_6 \quad (2)$$

To calculate the number of independent bulk 3-cycles we need the Betti number $b_3(T^6/\mathbb{Z}'_6)$ which is the dimension of the third homology group H_3 of the space. Because of the duality of the homology and cohomology groups we can as well compute the number of independent invariant 3-forms. Then

$$b_3 = b^3 = \sum_{p+q=3} b^{p,q} \quad (3)$$

where $b^{p,q}$ is the number of independent invariant p, q forms, *i.e.* with p holomorphic and q anti-holomorphic variables. For \mathbb{Z}'_6 , the invariant forms are $dz_1 \wedge dz_2 \wedge dz_3$, $dz_1 \wedge dz_2 \wedge d\bar{z}_3$ and their complex conjugates, so $b^{3,0} = 1 = b^{0,3}$ and $b^{2,1} = 1 = b^{1,2}$. Hence, the untwisted sector contributes

$$b_3^{(0)}(T^6/\mathbb{Z}'_6) = 4 \quad (4)$$

to the Betti number.

The point group action must be an automorphism of the lattice, so in $T^2_{1,2}$ we may take an $SU(3)$ lattice. Specifically we define the basis 1-cycles in $T^2_{1,2}$ by π_1 and $\pi_2 \equiv e^{i\pi/3}\pi_1$ in T^2_1 and π_3 and $\pi_4 \equiv e^{i\pi/3}\pi_3$ in T^2_2 . The orientation of $\pi_{1,3}$ relative to the real and imaginary axes of $z_{1,2}$ is arbitrary. Since θ acts as a reflection in T^2_3 the lattice, with basis 1-cycles π_5 and π_6 , is arbitrary. The point group action on the basis 1-cycles is then

$$\theta\pi_1 = \pi_2 \quad \text{and} \quad \theta\pi_2 = \pi_2 - \pi_1 \quad (5)$$

$$\theta\pi_3 = \pi_4 - \pi_3 \quad \text{and} \quad \theta\pi_4 = -\pi_3 \quad (6)$$

$$\theta\pi_5 = -\pi_5 \quad \text{and} \quad \theta\pi_6 = -\pi_6 \quad (7)$$

Now we construct a basis of invariant 3-cycles. With $\pi_{i,j,k} \equiv \pi_i \otimes \pi_j \otimes \pi_k$ where $i = 1, 2$, $j = 3, 4$, $k = 5, 6$ we define the \mathbb{Z}'_6 invariant 3-cycle

$$\begin{aligned} \rho_1 &\equiv (1 + \theta + \theta^2 + \theta^3 + \theta^4 + \theta^5)\pi_{1,3,5} \\ &= 2(1 + \theta + \theta^2)\pi_{1,3,5} \\ &= 2(\pi_{1,3,5} + \pi_{2,3,5} + \pi_{1,4,5} - 2\pi_{2,4,5}) \end{aligned} \quad (8)$$

In the same way

$$\begin{aligned} \rho_2 &\equiv 2(1 + \theta + \theta^2)\pi_{2,3,5} \\ &= 2(-\pi_{1,3,5} + 2\pi_{2,3,5} + 2\pi_{1,4,5} - \pi_{2,4,5}) \end{aligned} \quad (9)$$

and

$$\begin{aligned}\rho_3 &\equiv 2(1 + \theta + \theta^2)\pi_{2,4,5} \\ &= 2(-2\pi_{1,3,5} + \pi_{2,3,5} + \pi_{1,4,5} + \pi_{2,4,5})\end{aligned}\quad (10)$$

$$= \rho_2 - \rho_1 \quad (11)$$

Similarly, replacing $\pi_5 \rightarrow \pi_6$, we get

$$\rho_4 = 2(\pi_{1,3,6} + \pi_{2,3,6} + \pi_{1,4,6} - 2\pi_{2,4,6}) \quad (12)$$

$$\rho_5 = 2(-\pi_{1,3,6} + 2\pi_{2,3,6} + 2\pi_{1,4,6} - \pi_{2,4,6}) \quad (13)$$

$$\rho_6 = 2(-2\pi_{1,3,6} + \pi_{2,3,6} + \pi_{1,4,6} + \pi_{2,4,6}) \quad (14)$$

$$= \rho_5 - \rho_4 \quad (15)$$

Thus we can use the four cycles $\rho_1, \rho_3, \rho_4, \rho_6$ as the basis of the (untwisted) bulk 3-cycles, incidentally verifying (4). The most general invariant bulk 3-cycle is

$$\begin{aligned}2(1 + \theta + \theta^2) [(n_1\pi_1 + m_1\pi_2) \otimes (n_2\pi_3 + m_2\pi_4) \otimes (n_3\pi_5 + m_3\pi_6)] \\ = A_1\rho_1 + A_3\rho_3 + A_4\rho_4 + A_6\rho_6\end{aligned}\quad (16)$$

where (n_k, m_k) ($k = 1, 2, 3$) are the (coprime) wrapping numbers on the k th torus, and

$$A_1 = (n_1n_2 + n_1m_2 + m_1n_2)n_3 \quad (17)$$

$$A_3 = (m_1m_2 + n_1m_2 + m_1n_2)n_3 \quad (18)$$

$$A_4 = (n_1n_2 + n_1m_2 + m_1n_2)m_3 \quad (19)$$

$$A_6 = (m_1m_2 + n_1m_2 + m_1n_2)m_3 \quad (20)$$

The intersection number is defined as

$$\Pi_a \circ \Pi_b \equiv \frac{1}{6} \left(\sum_{i=0}^5 \theta^i \pi_a \right) \circ \left(\sum_{j=0}^5 \theta^j \pi_b \right) \quad (21)$$

which gives

$$\rho_1 \circ \rho_3 = 0 = \rho_4 \circ \rho_6 \quad (22)$$

$$\rho_1 \circ \rho_4 = -4, \quad \rho_1 \circ \rho_6 = 2 \quad (23)$$

$$\rho_3 \circ \rho_4 = 2, \quad \rho_3 \circ \rho_6 = -4 \quad (24)$$

In general, if π_a has wrapping numbers (n_k^a, m_k^a) ($k = 1, 2, 3$), and π_b has wrapping numbers (n_k^b, m_k^b) , then the intersection number of the orbifold-invariant 3-cycles Π_a and Π_b generated from π_a and π_b is

$$\begin{aligned}\Pi_a \circ \Pi_b &= -4(A_1^a A_4^b - A_4^a A_1^b) + 2(A_1^a A_6^b - A_6^a A_1^b) + 2(A_3^a A_4^b - A_4^a A_3^b) - \\ &\quad - 4(A_3^a A_6^b - A_6^a A_3^b) \equiv F(A_p^a, A_p^b)\end{aligned}\quad (25)$$

which is always even. Here A_p^a ($p = 1, 3, 4, 6$) relate to Π_a and are given by (17) - (20) with the wrapping numbers (n_k^a, m_k^a) , and similarly for the A_p^b which relate to Π_b .

Besides the (untwisted) bulk 3-cycles discussed above, there are also exceptional 3-cycles associated with (some of) the twisted sectors of the orbifold. They arise in twisted sectors in which there is a fixed torus, and consist of a collapsed 2-cycle at a fixed point times a 1-cycle in the invariant plane. For the \mathbb{Z}_6' orbifold the θ^2 and θ^4 twisted sectors leave T_3^2 invariant, and the θ^3 twisted sector leaves T_2^2 invariant.

In the θ^2 and θ^4 twisted sectors, there is a \mathbb{Z}_3 symmetry which has nine fixed points at

$$e_{i,j} \equiv \frac{i}{3}(e_1 + e_2) \otimes \frac{j}{3}(e_3 + e_4) \quad (26)$$

where $i, j = 0, 1, 2 \bmod 3$, e_1 and $e_2 \equiv e^{i\pi/3}e_1$ are the basis lattice vectors in T_1^2 , and e_3 and $e_4 \equiv e^{i\pi/3}e_3$ are the basis lattice vectors in T_2^2 . The \mathbb{Z}'_6 generator θ acts on the fixed points as

$$\theta \frac{i}{3}(e_1 + e_2) = -\frac{i}{3}(e_1 + e_2) \quad (27)$$

$$\theta \frac{j}{3}(e_3 + e_4) = \frac{j}{3}(e_3 + e_4) \quad (28)$$

Exceptional cycles in these sectors have the form $e_{i,j} \otimes (n_3\pi_5 + m_3\pi_6)$. The action of the point group is given by

$$\begin{aligned} \theta e_{i,j} \otimes (n_3\pi_5 + m_3\pi_6) &\equiv e_{\theta i, \theta j} \otimes \theta(n_3\pi_5 + m_3\pi_6) \\ &= e_{-i, j} \otimes (-n_3\pi_5 - m_3\pi_6) \end{aligned} \quad (29)$$

Thus there are six invariant 3-cycles in each θ^n ($n = 2, 4$) twisted sector. As a basis we may choose

$$\eta_j^{(n)} \equiv (e_{1,j}^{(n)} - e_{2,j}^{(n)}) \otimes \pi_5, \quad \tilde{\eta}_j^{(n)} \equiv (e_{1,j}^{(n)} - e_{2,j}^{(n)}) \otimes \pi_6 \quad (j = 0, 1, 2) \quad (30)$$

where $e_{i,j}^{(n)}$ is the collapsed 2-cycle at the fixed point $e_{i,j}$ in the θ^n twisted sector. The intersection numbers of these 2-cycles may be computed by blowing up the \mathbb{Z}_3 singularities [21]. This yields an intersection matrix which is the Cartan matrix $\mathbf{A}_{(2)}$ of the Lie algebra of $A_{(2)} \cong SU(3)$. Then, using the intersection numbers of the 1-cycles on T_3^2 , we find that

$$\eta_j^{(m)} \circ \tilde{\eta}_k^{(n)} = -\delta_{jk} A_{(2)}^{mn} \quad (31)$$

where

$$\mathbf{A}_{(2)} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad (32)$$

Exceptional cycles also occur in the θ^3 sector. There is a \mathbb{Z}_2 symmetry acting in T_1^2 and T_3^2 and this has sixteen fixed points at

$$f_{i,j} \equiv \frac{1}{2}(\sigma_1 e_1 + \sigma_2 e_2) \otimes \frac{1}{2}(\tau_1 e_5 + \tau_2 e_6) \quad (33)$$

where $\sigma_{1,2}, \tau_{1,2} = 0, 1 \pmod{2}$, and, using the notation of reference [19], $i, j = 1, 4, 5, 6$ correspond to the pairs (σ_1, σ_2) or (τ_1, τ_2)

$$1 \sim (0, 0), \quad 4 \sim (1, 0), \quad 5 \sim (0, 1), \quad 6 \sim (1, 1) \quad (34)$$

The \mathbb{Z}'_6 generator θ acts on the fixed points as

$$\theta \frac{1}{2}(\sigma_1 e_1 + \sigma_2 e_2) = \frac{1}{2}[-\sigma_2 e_1 + (\sigma_1 + \sigma_2)e_2] \quad (35)$$

$$\theta \frac{1}{2}(\tau_1 e_5 + \tau_2 e_6) = -\frac{1}{2}(\tau_1 e_5 + \tau_2 e_6) = \frac{1}{2}(\tau_1 e_5 + \tau_2 e_6) \quad (36)$$

so

$$\theta f_{1,j} = f_{1,j}, \quad \theta f_{4,j} = f_{5,j} \quad (37)$$

$$\theta f_{5,j} = f_{6,j}, \quad \theta f_{6,j} = f_{4,j} \quad (38)$$

The exceptional cycles are then $f_{i,j} \otimes (n_2\pi_3 + m_2\pi_4)$. Using (6) we construct the orbifold invariant exceptional 3-cycles:

$$(1 + \theta + \theta^2)f_{1,j} \otimes \pi_3 = 0 = (1 + \theta + \theta^2)f_{1,j} \otimes \pi_4 \quad (39)$$

$$(1 + \theta + \theta^2)f_{6,j} \otimes \pi_3 = (f_{6,j} - f_{4,j}) \otimes \pi_3 + (f_{4,j} - f_{5,j}) \otimes \pi_4 \equiv \epsilon_j \quad (40)$$

$$(1 + \theta + \theta^2)f_{4,j} \otimes \pi_3 = (f_{4,j} - f_{5,j}) \otimes \pi_3 + (f_{5,j} - f_{6,j}) \otimes \pi_4 \equiv \tilde{\epsilon}_j \quad (41)$$

$$\begin{aligned} (1 + \theta + \theta^2)f_{5,j} \otimes \pi_3 &= (f_{5,j} - f_{6,j}) \otimes \pi_3 + (f_{6,j} - f_{4,j}) \otimes \pi_4 \\ &= -\epsilon_j - \tilde{\epsilon}_j \end{aligned} \quad (42)$$

Fixed point \otimes 1-cycle	Exceptional 3-cycle
$f_{1,j} \otimes (n_2\pi_3 + m_2\pi_4)$	0
$f_{4,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$m_2\epsilon_j + (n_2 + m_2)\tilde{\epsilon}_j$
$f_{5,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$-(n_2 + m_2)\epsilon_j - n_2\tilde{\epsilon}_j$
$f_{6,j} \otimes (n_2\pi_3 + m_2\pi_4)$	$n_2\epsilon_j - m_2\tilde{\epsilon}_j$

Table 1: Relation between fixed points and exceptional 3-cycles.

It is easy to see that $f_{i,j} \otimes \pi_4$ generate the same orbifold invariants, so there are just eight independent orbifold-invariant exceptional 3-cycles ϵ_j and $\tilde{\epsilon}_j$ ($j = 1, 4, 5, 6$). The non-zero intersection numbers for the invariant combinations (40) and (41) are given by

$$\epsilon_j \circ \tilde{\epsilon}_k = -2\delta_{jk} \quad (43)$$

using the self-intersection number of -2 for an exceptional 2-cycle at the \mathbb{Z}_2 fixed point $f_{4,j}$ and the intersection numbers of the 1-cycles on T_2^2 . The relation between fixed points and exceptional cycles is shown in Table 1.

3 The \mathbb{Z}'_6 orientifold

We have already noted that the use of an orientifold is necessary if we are to obtain just the spectrum of the MSSM. In fact, like the D3-planes in the bottom-up models, the net cancellation of RR charge cannot be achieved using just D6-branes wrapping an orbifold. The use of an orientifold is also necessary for the cancellation of the RR charge. The embedding \mathcal{R} of the world-sheet parity operator Ω by an anti-holomorphic involution induces O6-planes that carry negative RR charge, and it is this that enables the necessary cancellation. The action of \mathcal{R} on the three complex coordinates z_k ($k = 1, 2, 3$) is

$$\mathcal{R}z_k = e^{i\phi_k}\bar{z}_k \quad (44)$$

where ϕ_k is an arbitrary phase that we may choose to be zero. Thus, \mathcal{R} acts as complex conjugation, and we require that this too is an automorphism of the lattice. This fixes the orientation of the basis 1-cycles in each torus relative to the $\text{Re}z_k$ axis. It requires them to be in one of two configurations **A** or **B**.

For $k = 1, 2$

$$\mathbf{A} : \quad \mathcal{R}\pi_{2k-1} = \pi_{2k-1}, \quad \mathcal{R}\pi_{2k} = \pi_{2k-1} - \pi_{2k} \quad (45)$$

$$\mathbf{B} : \quad \mathcal{R}\pi_{2k-1} = \pi_{2k}, \quad \mathcal{R}\pi_{2k} = \pi_{2k-1} \quad (46)$$

In either case, the complex structure U_k is given by

$$U_k \equiv \frac{\pi_{2k}}{\pi_{2k-1}} = e^{i\pi/3} = \frac{1}{2} + i\frac{\sqrt{3}}{2} \quad (47)$$

The fundamental tori in the two configurations are shown in Figs 1, 2; $\pi_{2k-1,2k}$ wraps $e_{2k-1,2k}$ respectively. On T_3^2 we have

$$\mathbf{A} : \mathcal{R}\pi_5 = \pi_5 \quad \text{and} \quad \mathcal{R}\pi_6 = -\pi_6 \quad (48)$$

$$\mathbf{B} : \mathcal{R}\pi_5 = \pi_5 \quad \text{and} \quad \mathcal{R}\pi_6 = \pi_5 - \pi_6 \quad (49)$$

In both cases π_5 is real, and in the **A** case π_6 is pure imaginary. Thus

$$U_3^{\mathbf{A}} = i\frac{R_6}{R_5} \quad (50)$$

where $R_{5,6}$ are the length of the 1-cycles $\pi_{5,6}$. In the **B** case, $\pi_6 + \mathcal{R}\pi_6 = \pi_5$, so that

$$U_3^{\mathbf{B}} = \frac{1}{2} + i\sqrt{\frac{R_6^2}{R_5^2} - \frac{1}{4}} \quad (51)$$

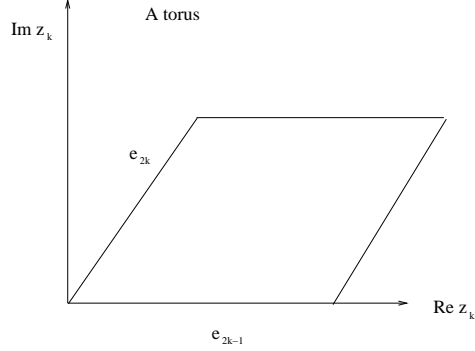


Figure 1: The fundamental tori T_k^2 , $k = 1, 2$ in the **A** configuration.

Lattice	$\mathcal{R}\rho_1$	$\mathcal{R}\rho_3$	$\mathcal{R}\rho_4$	$\mathcal{R}\rho_6$
AAA	ρ_1	$-\rho_1 - \rho_3$	$-\rho_4$	$\rho_4 + \rho_6$
AAB	ρ_1	$-\rho_1 - \rho_3$	$\rho_1 - \rho_4$	$-\rho_1 - \rho_3 + \rho_4 + \rho_6$
ABA and BAA	$\rho_1 + \rho_3$	$-\rho_3$	$-\rho_4 - \rho_6$	ρ_6
ABB and BAB	$\rho_1 + \rho_3$	$-\rho_3$	$\rho_1 + \rho_3 - \rho_4 - \rho_6$	$\rho_6 - \rho_3$
BBA	ρ_3	ρ_1	$-\rho_6$	$-\rho_4$
BBB	ρ_3	ρ_1	$\rho_3 - \rho_6$	$\rho_1 - \rho_4$

Table 2: \mathcal{R} -images of the bulk 3-cycles.

Both are summarised in the formula

$$U_3 = b_3 + i\sqrt{\frac{R_6^2}{R_5^2} - b_3^2} \quad (52)$$

where $b_3 = 0, \frac{1}{2}$ respectively for the **A**, **B** orientations. The fundamental tori in the two configurations are shown in Figs 3, 4; $\pi_{5,6}$ wrap $e_{5,6}$ respectively,

The \mathcal{R} -images of the four basis bulk 3-cycles $\rho_{1,3,4,6}$, defined in (8),(10),(12) and (14), on each of the lattices may now be calculated. They are given in Table 2. The O6-planes that are invariant under \mathcal{R} may then be identified. In each case there are two linearly independent \mathcal{R} -invariant combinations, which may be chosen to be those given in Table 3. The cancellation of RR tadpoles requires that the overall

AAA	ρ_1	$\rho_4 + 2\rho_6$
AAB	ρ_1	$-\rho_3 + \rho_4 + 2\rho_6$
ABA and BAA	$2\rho_1 + \rho_3$	ρ_6
ABB and BAB	$2\rho_1 + \rho_3$	$-\rho_3 + 2\rho_6$
BBA	$\rho_1 + \rho_3$	$\rho_4 - \rho_6$
BBB	$\rho_1 + \rho_3$	$\rho_1 - \rho_3 - 2\rho_4 + 2\rho_6$

Table 3: \mathcal{R} -invariant bulk 3-cycles.

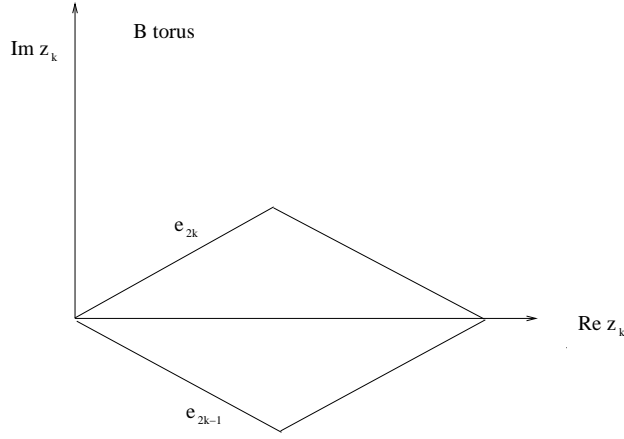


Figure 2: The fundamental tori T_k^2 , $k = 1, 2$ in the **B** configuration.

Lattice	Invariant	T_1^2	T_2^2	T_3^2
A	\mathcal{R}	π_1	π_3	π_5
	$\theta\mathcal{R}$	$\pi_1 + \pi_2$	$\pi_3 + \pi_4$	π_6
B	\mathcal{R}	$\pi_1 + \pi_2$	$\pi_3 + \pi_4$	π_5
	$\theta\mathcal{R}$	π_2	$\pi_3 - 2\pi_4$	$\pi_5 - 2\pi_6$

Table 4: \mathcal{R} - and $\theta\mathcal{R}$ -invariant cycles.

homology class of the D6-branes and O6-planes vanishes:

$$\sum_a N_a (\Pi_a + \Pi'_a) - 4\Pi_{\text{O6}} = 0 \quad (53)$$

where N_a is the number of D6-branes in the stack a , Π'_a is the orientifold dual of Π_a

$$\Pi'_a \equiv \mathcal{R}\Pi_a \quad (54)$$

and Π_{O6} is the homology class of the O6-planes. To determine the last of these we must consider the factorisable 3-cycles in detail. The \mathcal{R} - and $\theta\mathcal{R}$ -invariant cycles on the three tori T_k^2 are as given in Table 4. These generate point-group invariant 3-cycles for each lattice, as given in Table 5. The total homology class is obtained by summing the two contributions. Thus RR tadpole cancellation (53) requires that

$$\mathbf{AAA} : \quad \sum_a N_a [(2A_1^a - A_3^a)\rho_1 + A_6^a(\rho_4 + 2\rho_6)] = 4(\rho_1 + \rho_4 + 2\rho_6) \quad (55)$$

$$\mathbf{AAB} : \quad \sum_a N_a [(2A_1^a - A_3^a + A_4^a - A_6^a)\rho_1 - A_6^a(\rho_3 - \rho_4 - 2\rho_6)] = 8(\rho_1 + \rho_3 - \rho_4 - 2\rho_6) \quad (56)$$

$$\mathbf{ABA} : \quad \sum_a N_a [A_1^a(2\rho_1 + \rho_3) + (2A_6^a - A_4^a)\rho_6] = 4(2\rho_1 + \rho_3 - 3\rho_6) \quad (57)$$

$$\mathbf{BAA} : \quad \sum_a N_a [A_1^a(2\rho_1 + \rho_3) + (2A_6^a - A_4^a)\rho_6] = 4(2\rho_1 + \rho_3 + \rho_6) \quad (58)$$

$$\mathbf{ABB} : \quad \sum_a N_a [(2A_1^a + A_4^a)\rho_1 + (A_1^a + A_4^a - A_6^a)\rho_3 + (2A_6^a - A_4^a)\rho_6] = 8(\rho_1 - \rho_3 + 3\rho_6) \quad (59)$$

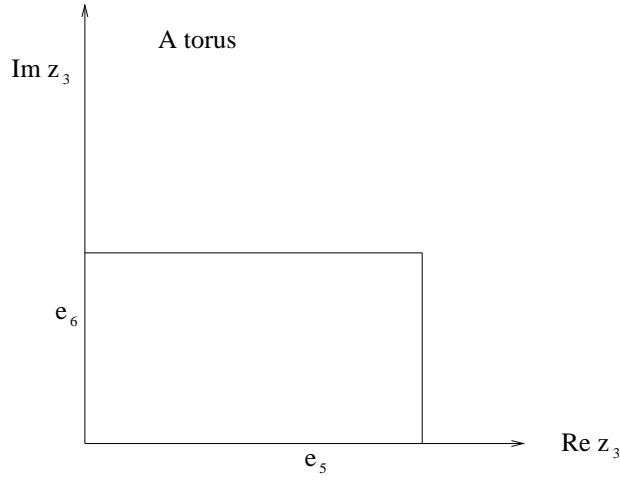


Figure 3: The fundamental torus T_3^2 , in the **A** configuration.

$$\mathbf{BAB} : \quad \sum_a N_a [(2A_1^a + A_4^a)\rho_1 + (A_1^a + A_4^a - A_6^a)\rho_3 + (2A_6^a - A_4^a)\rho_6] = 8(\rho_1 + \rho_3 - \rho_6) \quad (60)$$

$$\mathbf{BBA} : \quad \sum_a N_a [(A_1^a + A_3^a)(\rho_1 + \rho_3) + (A_4^a - A_6^a)(\rho_4 - \rho_6)] = 4(3\rho_1 + 3\rho_3 + \rho_4 + \rho_6) \quad (61)$$

$$\begin{aligned} \mathbf{BBB} : \quad \sum_a N_a [(A_1^a + A_3^a + A_6^a)\rho_1 + (A_1^a + A_3^a + A_4^a)\rho_3 + (A_4^a - A_6^a)(\rho_4 - \rho_6)] \\ = 8(2\rho_1 + \rho_3 - \rho_4 + \rho_6) \end{aligned} \quad (62)$$

where A_p^a ($p = 1, 3, 4, 6$) are the bulk coefficients for the stack a . In each case we obtain two constraints, there being two independent \mathcal{R} -invariant combinations of the four basis bulk 3-cycles. Note that, unlike in the \mathbb{Z}_6 case, the individual contributions from each stack a do not generally wrap the O6-plane.

The orientifold images of the exceptional cycles may also be computed. For the θ^2 - and θ^4 -sector exceptional cycles $\eta_j^{(n)}$ and $\tilde{\eta}_j^{(n)}$ with $n = 2, 4$ and $j = 0, 1, 2 \bmod 3$, defined in (30), they are given in Table 6 on all eight lattices. Similarly, for the θ^3 -sector exceptional cycles ϵ_j and $\tilde{\epsilon}_j$ with $j = 1, 4, 5, 6$, defined in (40) and (41), the orientifold images are given in Table 7 on all eight lattices. Note that on the **ABA** lattice the action of \mathcal{R} is just minus the action on the **BAA** lattice. Similarly for the **ABB** and **BAB** lattices.

The overall homology class for all exceptional branes and their orientifold images is required to vanish separately, since the O6-plane does not contribute. Thus

$$\sum_a N_a (\Pi_a^{\text{ex}} + \Pi_a^{\text{ex}'}) = 0 \quad (63)$$

We write the general exceptional brane (in the θ^3 -sector) in the form

$$\Pi_a^{\text{ex}} = \sum_{j=1,4,5,6} (\alpha_j^a \epsilon_j + \tilde{\alpha}_j^a \tilde{\epsilon}_j) \quad (64)$$

Using the results given in Table 7 the tadpole cancellation conditions (63) are:

$$\mathbf{AAA} : \quad \sum_a N_a (\alpha_j^a - 2\tilde{\alpha}_j^a) = 0 \quad (65)$$

$$\mathbf{AAB} : \quad \sum_a N_a (\alpha_p^a - 2\tilde{\alpha}_p^a) = 0, \quad \sum_a N_a \alpha_5^a = \sum_a N_a (\tilde{\alpha}_5^a + \tilde{\alpha}_6^a) = \sum_a N_a \alpha_6^a \quad (66)$$

Lattice	Invariant	$(n_1, m_1)(n_2, m_2)(n_3, m_3)$	3-cycle
AAA	\mathcal{R}	$(1, 0)(1, 0)(1, 0)$	ρ_1
	$\theta\mathcal{R}$	$(1, 1)(0, 1)(0, 1)$	$\rho_4 + 2\rho_6$
AAB	\mathcal{R}	$(1, 0)(1, 0)(1, 0)$	ρ_1
	$\theta\mathcal{R}$	$(1, 1)(0, 1)(1, -2)$	$\rho_1 + 2\rho_3 - 2\rho_4 - 4\rho_6$
ABA	\mathcal{R}	$(1, 0)(1, 1)(1, 0)$	$2\rho_1 + \rho_3$
	$\theta\mathcal{R}$	$(1, 1)(1, -2)(0, 1)$	$-3\rho_6$
BAA	\mathcal{R}	$(1, 1)(1, 0)(1, 0)$	$2\rho_1 + \rho_3$
	$\theta\mathcal{R}$	$(0, 1)(0, 1)(0, 1)$	ρ_6
ABB	\mathcal{R}	$(1, 0)(1, 1)(1, 0)$	$2\rho_1 + \rho_3$
	$\theta\mathcal{R}$	$(1, 1)(1, -2)(1, -2)$	$-3\rho_3 + 6\rho_6$
BAB	\mathcal{R}	$(1, 1)(1, 0)(1, 0)$	$2\rho_1 + \rho_3$
	$\theta\mathcal{R}$	$(0, 1)(0, 1)(1, -2)$	$\rho_3 - 2\rho_6$
BBA	\mathcal{R}	$(1, 1)(1, 1)(1, 0)$	$3\rho_1 + 3\rho_3$
	$\theta\mathcal{R}$	$(0, 1)(1, -2)(0, 1)$	$\rho_4 - \rho_6$
BBB	\mathcal{R}	$(1, 1)(1, 1)(1, 0)$	$3\rho_1 + 3\rho_3$
	$\theta\mathcal{R}$	$(0, 1)(1, -2)(1, -2)$	$\rho_1 - \rho_3 - 2\rho_4 + 2\rho_6$

Table 5: O6-planes of the \mathbb{Z}'_6 orientifold. The total homology class Π_{O6} is obtained by summing over the two orbits for each lattice.

Lattice	$\mathcal{R}\eta_j^{(n)}$	$\mathcal{R}\tilde{\eta}_j^{(n)}$
AAA	$-\eta_{-j}^{(n)}$	$\tilde{\eta}_{-j}^{(n)}$
AAB	$-\eta_{-j}^{(n)}$	$-\eta_{-j}^{(n)} + \tilde{\eta}_{-j}^{(n)}$
ABA	$-\eta_j^{(n)}$	$\tilde{\eta}_j^{(n)}$
BAA	$\eta_{-j}^{(n)}$	$-\tilde{\eta}_{-j}^{(n)}$
BBA	$\eta_j^{(n)}$	$-\tilde{\eta}_j^{(n)}$
BAB	$\eta_{-j}^{(n)}$	$\eta_{-j}^{(n)} - \tilde{\eta}_{-j}^{(n)}$
ABB	$-\eta_j^{(n)}$	$-\eta_j^{(n)} + \tilde{\eta}_j^{(n)}$
BBB	$\eta_j^{(n)}$	$\eta_j^{(n)} - \tilde{\eta}_j^{(n)}$

Table 6: \mathcal{R} -images of the θ^n -sector ($n = 2, 4$) exceptional 3-cycles. Note that the twist sector n is preserved [21] under the orientifold action \mathcal{R} .

Lattice	$\mathcal{R}\epsilon_1$	$\mathcal{R}\tilde{\epsilon}_1$	$\mathcal{R}\epsilon_4$	$\mathcal{R}\tilde{\epsilon}_4$	$\mathcal{R}\epsilon_5$	$\mathcal{R}\tilde{\epsilon}_5$	$\mathcal{R}\epsilon_6$	$\mathcal{R}\tilde{\epsilon}_6$
AAA	$-\epsilon_1 - \tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$-\epsilon_4 - \tilde{\epsilon}_4$	$\tilde{\epsilon}_4$	$-\epsilon_5 - \tilde{\epsilon}_5$	$\tilde{\epsilon}_5$	$-\epsilon_6 - \tilde{\epsilon}_6$	$\tilde{\epsilon}_6$
AAB	$-\epsilon_1 - \tilde{\epsilon}_1$	$\tilde{\epsilon}_1$	$-\epsilon_4 - \tilde{\epsilon}_4$	$\tilde{\epsilon}_4$	$-\epsilon_6 - \tilde{\epsilon}_6$	$\tilde{\epsilon}_6$	$-\epsilon_5 - \tilde{\epsilon}_5$	$\tilde{\epsilon}_5$
ABA	$-\epsilon_1$	$\epsilon_1 + \tilde{\epsilon}_1$	$-\epsilon_4$	$\epsilon_4 + \tilde{\epsilon}_4$	$-\epsilon_5$	$\epsilon_5 + \tilde{\epsilon}_5$	$-\epsilon_6$	$\epsilon_6 + \tilde{\epsilon}_6$
BAA	ϵ_1	$-\epsilon_1 - \tilde{\epsilon}_1$	ϵ_4	$-\epsilon_4 - \tilde{\epsilon}_4$	ϵ_5	$-\epsilon_5 - \tilde{\epsilon}_5$	ϵ_6	$-\epsilon_6 - \tilde{\epsilon}_6$
ABB	$-\epsilon_1$	$\epsilon_1 + \tilde{\epsilon}_1$	$-\epsilon_4$	$\epsilon_4 + \tilde{\epsilon}_4$	$-\epsilon_6$	$\epsilon_6 + \tilde{\epsilon}_6$	$-\epsilon_5$	$\epsilon_5 + \tilde{\epsilon}_5$
BAB	ϵ_1	$-\epsilon_1 - \tilde{\epsilon}_1$	ϵ_4	$-\epsilon_4 - \tilde{\epsilon}_4$	ϵ_6	$-\epsilon_6 - \tilde{\epsilon}_6$	ϵ_5	$-\epsilon_5 - \tilde{\epsilon}_5$
BBA	$-\tilde{\epsilon}_1$	$-\epsilon_1$	$-\tilde{\epsilon}_4$	$-\epsilon_4$	$-\tilde{\epsilon}_5$	$-\epsilon_5$	$-\tilde{\epsilon}_6$	$-\epsilon_6$
BBB	$-\tilde{\epsilon}_1$	$-\epsilon_1$	$-\tilde{\epsilon}_4$	$-\epsilon_4$	$-\tilde{\epsilon}_6$	$-\epsilon_6$	$-\tilde{\epsilon}_5$	$-\epsilon_5$

Table 7: \mathcal{R} -images of the θ^3 -sector exceptional 3-cycles.

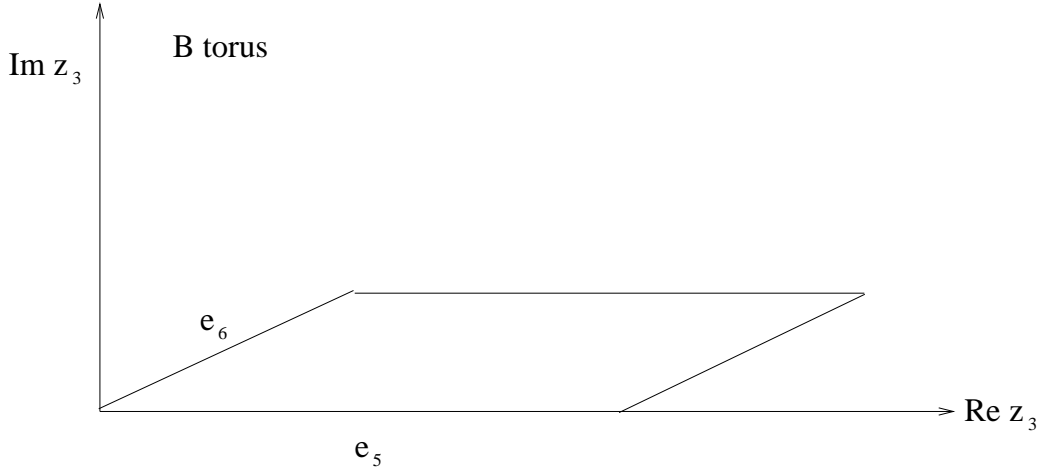


Figure 4: The fundamental torus T_3^2 in the **B** configuration.

$$\text{ABA} : \quad \sum_a N_a \tilde{\alpha}_j^a = 0 \quad (67)$$

$$\text{BAA} : \quad \sum_a N_a (2\alpha_j^a - \tilde{\alpha}_j^a) = 0 \quad (68)$$

$$\text{ABB} : \quad \sum_a N_a \tilde{\alpha}_p^a = 0, \quad \sum_a N_a \tilde{\alpha}_5^a = \sum_a N_a (\alpha_5^a - \alpha_6^a) = - \sum_a N_a \tilde{\alpha}_6^a \quad (69)$$

$$\text{BAB} : \quad \sum_a N_a (2\alpha_p^a - \tilde{\alpha}_p^a) = 0, \quad \sum_a N_a \tilde{\alpha}_5^a = \sum_a N_a (\alpha_5^a + \alpha_6^a) = \sum_a N_a \tilde{\alpha}_6^a \quad (70)$$

$$\text{BBA} : \quad \sum_a N_a (\alpha_j^a - \tilde{\alpha}_j^a) = 0 \quad (71)$$

$$\text{BBB} : \quad \sum_a N_a (\alpha_p^a - \tilde{\alpha}_p^a) = 0 = \sum_a N_a (\alpha_5^a - \tilde{\alpha}_6^a) = \sum_a N_a (\alpha_6^a - \tilde{\alpha}_5^a) \quad (72)$$

where $j = 1, 4, 5, 6$ and $p = 1, 4$. In each case there are 4 constraints, there being 4 independent \mathcal{R} -invariant combinations of the ϵ_j and $\tilde{\epsilon}_j$.

4 Supersymmetric bulk 3-cycles

The twist (2) ensures that the closed-string sector is supersymmetric. In order to avoid supersymmetry breaking in the open-string sector, the D6-branes must wrap special Lagrangian cycles. Then the stack a with wrapping numbers (n_k^a, m_k^a) ($k = 1, 2, 3$) is supersymmetric if

$$\sum_{k=1}^3 \phi_k^a = 0 \mod 2\pi \quad (73)$$

where ϕ_k^a is the angle that the 1-cycle in T_k^2 makes with the $\text{Re } z_k$ axis. Thus

$$\phi_k^a = \arg(n_k^a e_{2k-1} + m_k^a e_{2k}) \quad (74)$$

where e_{2k-1} and e_{2k} are complex numbers defining the basis 1-cycles in T_k^2 . Then, defining

$$Z^a \equiv \prod_{k=1}^3 e_{2k-1} (n_k^a + m_k^a U_k) \equiv X^a + iY^a \quad (75)$$

Lattice	X^a	Y^a
AAA	$2A_1^a - A_3^a - A_6^a \sqrt{3} \text{Im } U_3$	$\sqrt{3}A_3^a + (2A_4^a - A_6^a) \text{Im } U_3$
AAB	$2A_1^a - A_3^a + A_4^a - \frac{1}{2}A_6^a - A_6^a \sqrt{3} \text{Im } U_3$	$\sqrt{3}(A_3^a + \frac{1}{2}A_6^a) + (2A_4^a - A_6^a) \text{Im } U_3$
ABA and BAA	$\sqrt{3}A_1^a + (A_4^a - 2A_6^a) \text{Im } U_3$	$2A_3^a - A_1^a + A_4^a \sqrt{3} \text{Im } U_3$
ABB and BAB	$\sqrt{3}(A_1^a + \frac{1}{2}A_4^a) + (A_4^a - 2A_6^a) \text{Im } U_3$	$2A_3^a - A_1^a + A_6^a - \frac{1}{2}A_4^a + A_4^a \sqrt{3} \text{Im } U_3$
BBA	$A_1^a + A_3^a + (A_4^a - A_6^a) \sqrt{3} \text{Im } U_3$	$\sqrt{3}(A_3^a - A_1^a) + (A_4^a + A_6^a) \text{Im } U_3$
BBB	$(A_3^a + A_1^a + \frac{1}{2}A_6^a + \frac{1}{2}A_4^a) + (A_4 - A_6) \sqrt{3} \text{Im } U_3$	$\sqrt{3}(A_3^a - A_1^a + \frac{1}{2}A_6^a - \frac{1}{2}A_4^a) + (A_4 + A_6) \text{Im } U_3$

Table 8: The functions X^a and Y^a . (An overall positive factor of $R_1 R_3 R_5$ is omitted.) A stack a of D6-branes is supersymmetric if $X^a > 0$ and $Y^a = 0$.

where

$$U_k \equiv \frac{e_{2k}}{e_{2k-1}} \quad (76)$$

is the complex structure on T_k^2 , the condition (73) that a is supersymmetric may be written as

$$X^a > 0, Y^a = 0 \quad (77)$$

(A stack with $Y^a = 0$ but $X^a < 0$, so that $\sum_k \phi_k^a = \pi \text{ mod } 2\pi$, corresponds to a (supersymmetric) stack of anti-D-branes.) In our case $T_{1,2}^2$ are $SU(3)$ lattices and the values of U_k are given in (47) and (52). Thus

$$Z^a = e_1 e_3 e_5 [A_1^a - A_3^a + U_3(A_4^a - A_6^a) + e^{i\pi/3}(A_3^a + A_6^a U_3)] \quad (78)$$

where A_p^a ($p = 1, 3, 4, 6$) are the coefficients of the bulk 3-cycle defined in (17)...(20). If T_k^2 is of **A**-type, $e_{2k-1} = R_{2k-1} > 0$ is real and positive; in fact this is also the case for e_5 when T_3^2 is of **B**-type. However, when T_k^2 ($k = 1, 2$) is of **B**-type, $e_{2k-1} = R_{2k-1} e^{-i\pi/6}$. It is then straightforward to evaluate X^a and Y^a for the different lattices. The results are given in Table 8.

It follows that all of the \mathcal{R} -invariant bulk 3-cycles given in Table 3 satisfy $Y^a = 0$, and (with a suitable choice of overall sign) are therefore supersymmetric. However, unlike in the case of \mathbb{Z}_6 , there are supersymmetric 3-cycles that are not \mathcal{R} -invariant. In particular, there are supersymmetric 3-cycles that do not wrap the O6-planes. Thus, in general, $\Pi_a \circ \Pi_{\text{O6}} \neq 0$ for the \mathbb{Z}_6' orientifold. This has important implications which we will discuss shortly. Further, the tadpole cancellation conditions (55) - (62) allow an infinite number of solutions in this case. This is because the two independent conditions for each lattice involve independent linear combinations of the bulk coefficients A_p^a , whereas (for a fixed value of $\text{Im } U_3$) the positivity condition required by supersymmetry, $X^a > 0$ in (77), constrains just a single combination of these two; the orthogonal combination is unconstrained, and the condition $Y^a = 0$ involves an additional, independent combination of the bulk coefficients. The combination of the tadpole cancellation conditions corresponding to this unconstrained combination of the bulk coefficients may therefore be satisfied by cancelling positive against negative contributions from different stacks, thereby allowing an infinite number of solutions.

5 Fractional branes

The bulk 3-cycles (Π_a^{bulk}) or exceptional cycles (Π_a^{ex}) discussed in §§2,3 cannot be used alone for realistic phenomenology, since their intersection numbers are always even, as is apparent from (25),(31) and (43). To get odd intersection numbers, which, as noted in §1, is required if there is to be no unwanted additional vector-like matter, it is necessary to use fractional branes of the form

$$a = \frac{1}{2} \Pi_a^{\text{bulk}} + \frac{1}{2} \Pi_a^{\text{ex}} \quad (79)$$

$\Pi_a^{\text{bulk}} = \sum_p A_p \rho_p$ is associated with wrapping numbers $(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a)$, as shown in (16). The exceptional branes in the θ^3 sector are associated with the fixed points $f_{i,j}$, ($i, j = 1, 4, 5, 6$) in

(σ_1, σ_2)	$(0, 0) \text{ or } (\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, 0) \text{ or } (0, \frac{1}{2})$
(σ_5, σ_6)		
$(0, 0) \text{ or } (0, \frac{1}{2})$	$f_{11}, f_{15}, f_{61}, f_{65}$	$f_{41}, f_{45}, f_{51}, f_{55}$
$(\frac{1}{2}, 0) \text{ or } (\frac{1}{2}, \frac{1}{2})$	$f_{14}, f_{16}, f_{64}, f_{66}$	$f_{44}, f_{46}, f_{54}, f_{56}$

Table 9: The fixed points used to generate θ^3 -sector exceptional cycles in the case $(n_1^a, m_1^a)(n_3^a, m_3^a) = (1, 1)(0, 1) \bmod 2$.

$T_1^2 \otimes T_3^2$, as shown in (40) and (41). If Π_a^{bulk} is a supersymmetric bulk 3-cycle, then the fractional brane a , defined in (79), preserves supersymmetry provided that the exceptional part arises only from fixed points traversed by the bulk 3-cycle. In what follows we shall only consider fractional branes whose exceptional part Π_a^{ex} arises from the θ^3 -sector exceptional cycles ϵ_j and $\tilde{\epsilon}_j$ of the form (64). It appears to us that the θ^2 -sector exceptional cycles η_j and $\tilde{\eta}_j$, defined in (30), do not offer a rich enough structure to allow us to satisfy all of the constraints that we shall impose.

As an illustration of the stacks that we shall be considering, suppose that $(n_1^a, m_1^a)(n_3^a, m_3^a) = (1, 1)(0, 1) \bmod 2$. If the cycle on T_1^2 passes through any two lattice points, for example the origin and $n_1^a e_1 + m_1^a e_2$, then it wraps the \mathbb{Z}_2 fixed points 1 and 6, defined in (34). If the cycle on T_1^2 is offset from this situation by one half of one of the basis lattice vectors $(e_{1,2})$, for example if it passes through $\frac{1}{2}e_1$ and $(n_1^a + \frac{1}{2})e_1 + m_1^a e_2$, then it wraps the \mathbb{Z}_2 fixed points 4 and 5. However, if the cycle is offset by one half of both basis lattice vectors, for example if it passes through $\frac{1}{2}(e_1 + e_2)$ and $(n_1^a + \frac{1}{2})e_1 + (m_1^a + \frac{1}{2})e_2$, then it again wraps the fixed points 1 and 6. Similarly, depending on the offset, the cycle on T_3^2 may wrap 1 and 5, or 4 and 6. Writing the offset in the form $\sum_{i=1,2} \sigma_i e_i \otimes \sum_{j=5,6} \sigma_j e_j$ with $\sigma_{i,j} = 0, \frac{1}{2}$, the exceptional cycles involved are those associated with the fixed points given in Table 9. Obviously a similar analysis applies for other choices of $(n_1^a, m_1^a)(n_3^a, m_3^a) \bmod 2$.

In general, supersymmetry requires that the exceptional part Π_a^{ex} of a derives from four fixed points, $f_{i_1^a j_1^a}, f_{i_1^a j_2^a}, f_{i_2^a j_1^a}, f_{i_2^a j_2^a}$, with (i_1^a, i_2^a) the fixed points in T_1^2 , and (j_1^a, j_2^a) those in T_3^2 . Depending on the offset, there are two choices for (i_1^a, i_2^a) in T_1^2 , and similarly two choices for (j_1^a, j_2^a) in T_3^2 . Consequently there are a total of four sets of four fixed points, any one of which may be used to determine Π_a^{ex} while preserving supersymmetry. The choice of Wilson lines affects the relative signs with which the contributions from the four fixed points are combined to determine Π_a^{ex} . The rule is that

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \rightarrow f_{i_1^a j_1^a}, f_{i_1^a j_2^a}, f_{i_2^a j_1^a}, f_{i_2^a j_2^a} \quad (80)$$

$$\rightarrow (-1)^{\tau_0^a} [f_{i_1^a j_1^a} + (-1)^{\tau_2^a} f_{i_1^a j_2^a} + (-1)^{\tau_1^a} f_{i_2^a j_1^a} + (-1)^{\tau_1^a + \tau_2^a} f_{i_2^a j_2^a}] \quad (81)$$

where $\tau_{0,1,2}^a = 0, 1$ with $\tau_1^a = 1$ corresponding to a Wilson line in T_1^2 and likewise for τ_2^a in T_3^2 . The fixed point $f_{i,j}$ ($i, j = 1, 4, 5, 6$) with the 1-cycle $n_2^a \pi_3 + m_2^a \pi_4$ is then associated with (orbifold-invariant) exceptional cycle as shown in Table 1. Thus in the example above with $(n_1^a, m_1^a)(n_3^a, m_3^a) = (1, 1)(0, 1) \bmod 2$, if we take no offset in both planes, then $(i_1^a, i_2^a) = (16)$ and $(j_1^a, j_2^a) = (15)$. With this choice

$$\Pi_a^{\text{ex}}(16)(15)(n_2^a, m_2^a) = (-1)^{\tau_0^a + \tau_1^a} \{n_2^a [\epsilon_1 + (-1)^{\tau_2^a} \epsilon_5] - m_2^a [\tilde{\epsilon}_1 + (-1)^{\tau_2^a} \tilde{\epsilon}_5]\} \quad (82)$$

We noted earlier that given any two stacks a and b of D6-branes, then there is chiral matter whose spectrum is determined by the intersection numbers of the 3-cycles. There are $a \circ b$ chiral matter multiplets in the bifundamental representation $(\mathbf{N}_a, \bar{\mathbf{N}}_b)$ of $U(N_a) \otimes U(N_b)$, and $a \circ b'$ matter multiplets in the representation $(\mathbf{N}_a, \mathbf{N}_b)$, where $b' \equiv \mathcal{R}b$ is the orientifold image of b . In general, there is also chiral matter in the symmetric \mathbf{S}_a and antisymmetric representations \mathbf{A}_a of the gauge group $U(N_a)$, and likewise for $U(N_b)$, where the dimensions of these representations are

$$[\mathbf{S}_a] \equiv (\mathbf{N}_a \times \mathbf{N}_a)_{\text{symm}} = \frac{1}{2} N_a (N_a + 1) \quad (83)$$

$$[\mathbf{A}_a] \equiv (\mathbf{N}_a \times \mathbf{N}_a)_{\text{antisymm}} = \frac{1}{2} N_a (N_a - 1) \quad (84)$$

The number of multiplets in the \mathbf{S}_a representation is $\frac{1}{2}(a \circ a' - a \circ \Pi_{\text{O6}})$, and the number of multiplets in the \mathbf{A}_a representation is $\frac{1}{2}(a \circ a' + a \circ \Pi_{\text{O6}})$, where Π_{O6} is the total O6-brane homology class given in Table 5. If $a \circ \Pi_{\text{O6}} \neq 0$, then copies of one or both representations are inevitably present. This explains the importance of our observation at the end of §4 that this quantity is generally non-zero on the \mathbb{Z}'_6 orientifold. Of course, for a specific stack, it may happen that $a \circ \Pi_{\text{O6}} = 0$, in which case the absence of one of the representations ensures that neither representation is present. When $N_a = 3$, as required to get the $SU(3)_c$ gauge group of QCD, there will in general be chiral matter in the $\mathbf{S}_a = \mathbf{6}$ and $\mathbf{A}_a = \bar{\mathbf{3}}$ of the $SU(3)$. Similarly, when $N_b = 2$, as required to get the $SU(2)_L$ part of the electroweak gauge group, there will in general be chiral matter in the $\mathbf{S}_b = \mathbf{3}$ and $\mathbf{A}_a = \bar{\mathbf{1}}$ of $SU(2)$. For both groups the appearance of the symmetric representation is excluded phenomenologically. Quark singlet states q_L^c , in the $\bar{\mathbf{3}}$ representation of $SU(3)_c$, do occur of course, and similarly for lepton singlet states ℓ_L^c in the $\bar{\mathbf{1}}$ representation of $SU(2)_L$. Thus, we must exclude the appearance of the representations \mathbf{S}_a and \mathbf{S}_b , but not necessarily of \mathbf{A}_a or \mathbf{A}_b . Consequently, we impose the constraints

$$a \circ a' = a \circ \Pi_{\text{O6}} \quad (85)$$

$$b \circ b' = b \circ \Pi_{\text{O6}} \quad (86)$$

With these constraints the number of multiplets in the antisymmetric representation \mathbf{A}_a is then $a \circ \Pi_{\text{O6}}$, so we require too that

$$|a \circ \Pi_{\text{O6}}| \leq 3 \quad (87)$$

since otherwise there would again be non-minimal vector-like quark singlet matter. Similarly we require that

$$|b \circ \Pi_{\text{O6}}| \leq 3 \quad (88)$$

to avoid unwanted vector-like lepton singlets.

For fractional branes of the form (79),

$$a \circ a' = \frac{1}{4}\Pi_a^{\text{bulk}} \circ \Pi_a^{\text{bulk}'} + \frac{1}{4}\Pi_a^{\text{ex}} \circ \Pi_a^{\text{ex}'} \quad (89)$$

and

$$a \circ \Pi_{\text{O6}} = \frac{1}{2}\Pi_a^{\text{bulk}} \circ \Pi_{\text{O6}} \quad (90)$$

since there is no intersection between the O6-planes and the exceptional branes. For given wrapping numbers $(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a)$, the form (16) of the bulk part Π_a^{bulk} of a is determined by the formulae (17)...(20) for the bulk coefficients A_p^a ($p = 1, 3, 4, 6$). For any given lattice, it is then straightforward to calculate $\Pi_a^{\text{bulk}'}$ using Table 2 and hence, using (25), to calculate

$$f(A_p^a) \equiv \Pi_a^{\text{bulk}} \circ \Pi_a^{\text{bulk}'} = F(A_p^a, A_p^{a'}) \quad (91)$$

where $A_p^a, A_p^{a'}$ are respectively the bulk coefficients for $\Pi_a^{\text{bulk}}, \Pi_a^{\text{bulk}'}$. Similarly, using Table 5, it is straightforward to calculate

$$g(A_p^a) \equiv \Pi_a^{\text{bulk}} \circ \Pi_{\text{O6}} = F(A_p^a, S_p) \quad (92)$$

where S_p are the bulk coefficients of the O6-planes. The calculation of

$$\phi(n_2^a, m_2^a) \equiv \Pi_a^{\text{ex}} \circ \Pi_a^{\text{ex}'} \quad (93)$$

using (43) is equally straightforward, but also depends upon which one of the four sets of four fixed points $(i_1^a, i_2^a)(j_1^a, j_2^a)$ is chosen. The constraint (85) may then be written as

$$f(A_p^a) + \phi(n_2^a, m_2^a) = 2g(A_p^a) \quad (94)$$

and (87) as

$$|g(A_p^a)| \leq 6 \quad (95)$$

Similarly, for the stack b we require that

$$f(A_p^b) + \phi(n_2^b, m_2^b) = 2g(A_p^b) \quad (96)$$

and

$$|g(A_p^b)| \leq 6 \quad (97)$$

Each pair of wrapping numbers (n_k^a, m_k^a) ($k = 1, 2, 3$) may take one of the three values $(1, 1), (1, 0), (0, 1) \bmod 2$, since n_k^a and m_k^a are coprime. Thus, in general there are 27 possibilities for $(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a) \bmod 2$. However, from (5), (6) and (7), it follows that the \mathbb{Z}'_6 point group generator θ acts on the wrapping numbers as

$$\theta(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a) = (-m_1^a, n_1^a + m_1^a)(-n_2^a - m_2^a, n_2^a)(-n_3^a, -m_3^a) \quad (98)$$

This reduces the 27 possibilities to 9, since any choice is related to two others by the action of θ . Since (n_3^a, m_3^a) is invariant mod 2 under the action of θ , we may choose a “gauge” in which the representative element in each class has $(n_1^a, m_1^a) = (n_3^a, m_3^a) \bmod 2$. The 9 classes may be reduced further by including the action \mathcal{R} of the world sheet parity operator, but the action depends upon the lattice choice. An important difference arises between the four lattices in which T_3^2 is **A** type, and the four in which it is of **B** type. In the former case, it follows from (48) that all 3 choices for $(n_3^a, m_3^a) = (1, 1), (1, 0), (0, 1) \bmod 2$ are \mathcal{R} invariant. This means that the 3 classes defined above are not mixed by the action of \mathcal{R} . The 3 elements within each class split into $1 + 2$, where 1 is \mathcal{R} invariant and the 2 are related under the action of \mathcal{R} . Thus, on the 4 lattices with T_3^2 of **A** type, the 3 classes of 3 reduce to 3 classes of 2, leaving 6 in total. In the latter case, when T_3^2 is of **B** type, it follows from (49) that $(n_3^a, m_3^a) = (1, 0) \bmod 2$ is \mathcal{R} invariant, and that $(1, 1) \leftrightarrow (0, 1)$ under the action of \mathcal{R} . This means that of the 9 classes there is just one that is \mathcal{R} invariant, and the remaining 8 split into 4 classes of 2 in which each pair is related by the action of \mathcal{R} . Thus the 9 classes reduce to 5. The results are summarised in Table 10. The functions $f(A_p^a)$ and $g(A_p^a)$ are summarised in Table 11 and the function $\phi(n_2^a, m_2^a)$ is given in Table 12. Their values (mod 8) are given in Table 10. By inspection we can see that the requirement (94) that there are no symmetric representations places no further restrictions (mod 8) on the allowed wrapping numbers when T_3^2 is of **A** type. However, when T_3^2 is of **B** type, (94) limits the possible wrapping numbers to just 2 of the 5 classes allowed hitherto.

As noted earlier, the function $\phi(n_2^a, m_2^a)$ defined in (93) depends upon which pairs of fixed points are used in T_1^2 and T_3^2 , although we have not displayed this dependence. Table 12 gives the results when the (non-offset) pair (16) is chosen in both tori in the $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 1) \bmod 2$ case, when (14) is chosen in the $(1, 0)$ case, and when (15) is chosen on the $(0, 1)$ case. The modifications that arise when the other pair of fixed points, the offset pair, is chosen in one or both tori may be summarised as follows. For the four lattices **AAA**, **ABA**, **BAA**, **BBA** in which T_3^2 is of **A** type, there is no modification when the offset pair is chosen in T_3^2 , but an additional multiplicative factor of $2(-1)^{\tau_1^a} - 1$ is to be inserted when the offset pair is used in T_1^2 . For the four lattices **AAB**, **ABB**, **BAB**, **BBB** in which T_3^2 is of **B** type, the four different choices of pairs of fixed points can lead to different results. When the offset pair is used in T_3^2 but not in T_1^2 , a multiplicative factor of $(-1)^{\tau_2^a}$ is to be inserted, but *only* in the case $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0) \bmod 2$. When the offset pair is used in T_1^2 but not in T_3^2 , a multiplicative factor of $2(-1)^{\tau_1^a} - 1$ is to be inserted in all cases. When the offset pair is used in both T_1^2 and T_3^2 , both factors are to be inserted. Since $2(-1)^{\tau_1^a} - 1$ and $(-1)^{\tau_2^a}$ are both odd, their inclusion does not affect our conclusion that when T_3^2 is of **A** type the allowed wrapping numbers are not further restricted (mod 8) by equation (94). In other words, for these lattices, the allowed classes of wrapping numbers are unaffected by the inclusion of Wilson lines.

However, when T_3^2 is of **B** type, the allowed classes of wrapping numbers could be affected by Wilson lines. A difference could arise whenever $\phi(n_2^a, m_2^a) = \pm 2 \bmod 8$. Insertion of a factor $2(-1)^{\tau_1^a} - 1$ has no effect since $2 = -6 \bmod 8$, but insertion of a factor $(-1)^{\tau_2^a}$ clearly *does* have an effect. However, $\phi(n_2^a, m_2^a) = \pm 2 \bmod 8$ only occurs when $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 1) \bmod 2$, and the factor of $(-1)^{\tau_2^a}$ does not then occur, as noted above. Thus, for the four lattices **AAB**, **ABB**, **BAB**, **BBB** in which T_3^2 is of **B** type also, the (two) allowed classes of wrapping numbers are unaffected by the inclusion of Wilson lines. In this case, there are 3 possibilities for a and b . In two of them, a and b are in the

Lattice	$(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a) \bmod 2$	$f(A_i)$	$\phi(n_2^a, m_2^a)$	$2g(A_i)$
AAA	$(1, 1)(1, 1)(1, 1) \leftrightarrow (1, 1)(1, 0)(1, 1)$	4	4	0
	$(1, 1)(0, 1)(1, 1)$	0	0	0
	$(1, 0)(1, 1)(1, 0) \leftrightarrow (1, 0)(0, 1)(1, 0)$	0	4	4
	$(1, 0)(1, 0)(1, 0)$	0	0	0
	$(0, 1)(1, 0)(0, 1) \leftrightarrow (0, 1)(0, 1)(0, 1)$	0	4	4
	$(0, 1)(1, 1)(0, 1)$	0	0	0
BAA and ABA	$(1, 1)(1, 1)(1, 1) \leftrightarrow (1, 1)(0, 1)(1, 1)$	4	4	0
	$(1, 1)(1, 0)(1, 1)$	0	0	0
	$(1, 0)(1, 0)(1, 0) \leftrightarrow (1, 0)(0, 1)(1, 0)$	0	4	4
	$(1, 0)(1, 1)(1, 0)$	0	0	0
	$(0, 1)(1, 1)(0, 1) \leftrightarrow (0, 1)(1, 0)(0, 1)$	0	4	4
	$(0, 1)(0, 1)(0, 1)$	0	0	0
BBA	$(1, 1)(1, 0)(1, 1) \leftrightarrow (1, 1)(0, 1)(1, 1)$	4	4	0
	$(1, 1)(1, 1)(1, 1)$	0	0	0
	$(1, 0)(1, 1)(1, 0) \leftrightarrow (1, 0)(1, 0)(1, 0)$	0	4	4
	$(1, 0)(0, 1)(1, 0)$	0	0	0
	$(0, 1)(1, 1)(0, 1) \leftrightarrow (0, 1)(0, 1)(0, 1)$	0	4	4
	$(0, 1)(1, 0)(0, 1)$	0	0	0
AAB	$(1, 1)(1, 1)(1, 1) \leftrightarrow (0, 1)(0, 1)(0, 1)$	2	6	0
	$(1, 1)(1, 0)(1, 1) \leftrightarrow (0, 1)(1, 0)(0, 1)$	2	2	0
	$(1, 1)(0, 1)(1, 1) \leftrightarrow (0, 1)(1, 1)(0, 1)$	4	0	0
	$(1, 0)(1, 1)(1, 0) \leftrightarrow (1, 0)(0, 1)(1, 0)$	0	4	0
	$(1, 0)(1, 0)(1, 0)$	0	0	0
BAB (and ABB)	$(1, 1)(1, 1)(1, 1) \leftrightarrow (0, 1)(1, 1)(0, 1)$	6	2(6)	0
	$(1, 1)(1, 0)(1, 1) \leftrightarrow (0, 1)(0, 1)(0, 1)$	4	0	0
	$(1, 1)(0, 1)(1, 1) \leftrightarrow (0, 1)(1, 0)(0, 1)$	6	6(2)	0
	$(1, 0)(1, 0)(1, 0) \leftrightarrow (1, 0)(0, 1)(1, 0)$	0	4	0
	$(1, 0)(1, 1)(1, 0)$	0	0	0
BBB	$(1, 1)(1, 1)(1, 1) \leftrightarrow (0, 1)(1, 0)(0, 1)$	4	0	0
	$(1, 1)(1, 0)(1, 1) \leftrightarrow (0, 1)(1, 1)(0, 1)$	2	2	0
	$(1, 1)(0, 1)(1, 1) \leftrightarrow (0, 1)(0, 1)(0, 1)$	2	6	0
	$(1, 0)(1, 1)(1, 0) \leftrightarrow (1, 0)(1, 0)(1, 0)$	0	4	0
	$(1, 0)(0, 1)(1, 0)$	0	0	0

Table 10: Inequivalent wrapping number classes $(n_k^a, m_k^a) \bmod 2$ for the various lattices, and the values of the functions $f(A_p)$, $\phi(n_2^a, m_2^a)$ and $2g(A_p) \bmod 8$.

Lattice	$f(A_p^a)$	$g(A_p^a)$
AAA	$4(2A_1^a A_4^a - 2A_1^a A_6^a - A_3^a A_6^a)$	$2(2A_4^a - 3A_3^a - A_6^a)$
AAB	$4(2A_1^a A_4^a - 2A_1^a A_6^a - A_3^a A_6^a) + 2(2A_4^{a2} - 2A_4^a A_6^a - A_6^{a2})$	$4(3A_3^a + A_4^a + A_6^a)$
ABA	$4(A_1^a A_4^a + 2A_1^a A_6^a - 2A_3^a A_6^a)$	$6(2A_3^a + A_4^a - A_1^a)$
BAA	$4(A_1^a A_4^a + 2A_1^a A_6^a - 2A_3^a A_6^a)$	$2(A_1^a - 2A_3^a + A_4^a)$
ABB	$4(A_1^a A_4^a + 2A_1^a A_6^a - 2A_3^a A_6^a) + 2(A_4^{a2} + 2A_4^a A_6^a - 2A_6^{a2})$	$12(A_1^a - 2A_3^a + A_4^a - A_6^a)$
BAB	$4(A_1^a A_4^a + 2A_1^a A_6^a - 2A_3^a A_6^a) + 2(A_4^{a2} + 2A_4^a A_6^a - 2A_6^{a2})$	$4(-A_1^a + 2A_3^a + A_4^a + A_6^a)$
BBA	$4(4A_1^a A_6^a - A_1^a A_4^a - A_3^a A_6^a)$	$6(A_3^a - A_1^a + A_4^a + A_6^a)$
BBB	$-4(4A_1^a A_4^a - 4A_1^a A_6^a + A_3^a A_6^a) + 2(-A_4^{a2} + 4A_4^a A_6^a - A_6^{a2})$	$12(A_1^a - A_3^a + A_4^a)$

Table 11: The functions $f(A_p^a)$ and $g(A_p^a)$ for various lattices.

Lattice	(1, 1)	(1, 0)	(0, 1)
AAA	$4n_2^a(n_2^a + 2m_2^a)$	$-4m_2^a(m_2^a + 2n_2^a)$	$4(m_2^{a^2} - n_2^{a^2})$
AAB	$2n_2^a(n_2^a + 2m_2^a)$	$-4m_2^a(m_2^a + 2n_2^a)$	$2(m_2^{a^2} - n_2^{a^2})$
ABA	$4m_2^a(m_2^a + 2n_2^a)$	$-4(m_2^{a^2} - n_2^{a^2})$	$-4n_2^a(n_2^a + 2m_2^a)$
BAA	$-4m_2^a(m_2^a + 2n_2^a)$	$4(m_2^{a^2} - n_2^{a^2})$	$4n_2^a(n_2^a + 2m_2^a)$
ABB	$2m_2^a(m_2^a + 2n_2^a)$	$-4(m_2^{a^2} - n_2^{a^2})$	$-2n_2^a(n_2^a + 2m_2^a)$
BAB	$-2m_2^a(m_2^a + 2n_2^a)$	$4(m_2^{a^2} - n_2^{a^2})$	$2n_2^a(n_2^a + 2m_2^a)$
BBA	$-4(m_2^{a^2} - n_2^{a^2})$	$-4n_2^a(n_2^a + 2m_2^a)$	$4m_2^a(m_2^a + 2n_2^a)$
BBB	$-2(m_2^{a^2} - n_2^{a^2})$	$-4n_2^a(n_2^a + 2m_2^a)$	$2m_2^a(m_2^a + 2n_2^a)$

Table 12: The function $\phi(n_2^a, m_2^a)$ for various lattices. The three values for ϕ correspond to the three values of $(n_1^a, m_1^a) = (n_3^a, m_3^a) \bmod 2$ when the singular points chosen in both tori include the origin.

Lattice	$f_{AB'}(A_p^a, A_p^b)$
AAA	$4(A_1^a A_4^b + A_4^a A_1^b) - 2(A_1^a A_6^b + A_6^a A_1^b) - 2(A_3^a A_4^b + A_4^a A_3^b) - 2(A_3^a A_6^b + A_6^a A_3^b)$
ABA and BAA	$2(A_1^a A_4^b + A_4^a A_1^b) + 2(A_1^a A_6^b + A_6^a A_1^b) + 2(A_3^a A_4^b + A_4^a A_3^b) - 4(A_3^a A_6^b + A_6^a A_3^b)$
BBA	$-2(A_1^a A_4^b + A_4^a A_1^b) + 4(A_1^a A_6^b + A_6^a A_1^b) + 4(A_3^a A_4^b + A_4^a A_3^b) - 2(A_3^a A_6^b + A_6^a A_3^b)$
AAB	$4(A_1^a A_4^b + A_4^a A_1^b) + 4A_4^a A_4^b - 2(A_1^a A_6^b + A_6^a A_1^b) - 2(A_3^a A_4^b + A_4^a A_3^b) - 2(A_3^a A_6^b + A_6^a A_3^b) - 2A_6^a A_6^b - 2(A_4^a A_6^b + A_6^a A_4^b)$
ABB and AAA	$2(A_1^a A_4^b + A_4^a A_1^b) + 2A_4^a A_4^b + 2(A_1^a A_6^b + A_6^a A_1^b) + 2(A_3^a A_4^b + A_4^a A_3^b) - 4(A_3^a A_6^b + A_6^a A_3^b) - 4A_6^a A_6^b + 2(A_4^a A_6^b + A_6^a A_4^b)$
BBB	$-2(A_1^a A_4^b + A_4^a A_1^b) - 2A_4^a A_4^b + 4(A_1^a A_6^b + A_6^a A_1^b) + 4(A_3^a A_4^b + A_4^a A_3^b) - 2(A_3^a A_6^b + A_6^a A_3^b) - 2A_6^a A_6^b + 4(A_4^a A_6^b + A_6^a A_4^b)$

Table 13: The function $f_{AB'}$, defined in (100), for the various lattices.

same class; in the third, they are in different classes. In the last of these possibilities, it is convenient to take the stack a to be that in which $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0) \bmod 2$ and $(n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$. Consequently we may no longer assume that $N_a = 3$ and $N_b = 2$, as we have done hitherto.

In this paper we see whether we can construct realistic models, that is models in which the intersection numbers $(|a \circ b|, |a \circ b'|) = (2, 1)$ or $(1, 2)$, using the 3 possible combinations of wrapping numbers for each of the 4 lattices in which T_3^2 is of **B** type. As noted in the introduction, if $a \circ b$ and $a \circ b'$ have the same sign, then $N_a = 3$ and $N_b = 2$. If not, $N_a = 2$ and $N_b = 3$. The contributions from the bulk parts to $a \circ b$ and $a \circ b'$ are determined by the functions f_{AB} and $f_{AB'}$ respectively. The former is independent of the lattice and is given by

$$f_{AB} \equiv \Pi_a^{\text{bulk}} \circ \Pi_b^{\text{bulk}} = F(A_p^a, A_p^b) \quad (99)$$

where $F(A_p^a, A_p^b)$ is defined in (25). The latter,

$$f_{AB'} \equiv \Pi_a^{\text{bulk}} \circ \Pi_b^{\text{bulk}'} = F(A_p^a, A_p^{b'}) \quad (100)$$

however, is lattice-dependent. Table 13 gives the function $f_{AB'}$ for each lattice. The contribution to $a \circ b$ and $a \circ b'$ from the exceptional parts depends upon the fixed points chosen for Π_a^{ex} and Π_b^{ex} . Since there are four possible sets of fixed points for each exceptional part, there are, in principle 10 or 16 combinations to consider, depending upon whether a and b have the same or different sets of wrapping numbers (mod 2). The required intersection numbers are then

$$a \circ b = \frac{1}{4} f_{AB} + \frac{1}{4} \Pi_a^{\text{ex}}(i_1^a, i_2^a)(j_1^a, j_2^a)(n_2^a, m_2^a) \circ \Pi_b^{\text{ex}}(i_1^b, i_2^b)(j_1^b, j_2^b)(n_2^b, m_2^b) \quad (101)$$

$$a \circ b' = \frac{1}{4} f_{AB'} + \frac{1}{4} \Pi_a^{\text{ex}}(i_1^a, i_2^a)(j_1^a, j_2^a)(n_2^a, m_2^a) \circ \Pi_b^{\text{ex}}(i_1^b, i_2^b)(j_1^b, j_2^b)'(n_2^b, m_2^b) \quad (102)$$

where we are using the notation of equation (82). In what follows we abbreviate this notation by

$$\Pi_a^{\text{ex}}(i_1^a, i_2^a)(j_1^a, j_2^a)(n_2^a, m_2^a) \rightarrow (i_1^a, i_2^a)(j_1^a, j_2^a) \quad (103)$$

$$\Pi_a^{\text{ex}}(i_1^a, i_2^a)(j_1^a, j_2^a)'(n_2^a, m_2^a) \rightarrow (i_1^a, i_2^a)(j_1^a, j_2^a)' \quad (104)$$

Like the contribution f_{AB} from the bulk branes to $a \circ b$, the contribution from the exceptional branes is independent of the lattice, and depends only on the associated fixed points. We have seen that on all four lattices in which T_3^2 is of **B** type, the requirement that there is no matter in symmetric representations of the gauge group means that we must consider three cases: (i) $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 1) \bmod 2$, (ii) $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 0) \bmod 2$, and (iii) $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0) \bmod 2$, $(n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$. Thus exceptional branes associated with the same fixed points arise on all four lattices. In the next section we give the (universal) results for the contributions of these exceptional branes to $a \circ b$. (As is apparent from Table 10, the allowed values of $(n_2^{a,b}, m_2^{a,b}) \bmod 2$ are lattice-dependent, but we present the results for arbitrary values of $(n_2^{a,b}, m_2^{a,b})$.)

The corresponding contributions from the exceptional branes to $a \circ b'$ are presented for each lattice in the appendices. In all cases we give the results for the case that (i_1^a, i_2^a) and (i_1^b, i_2^b) are the offset pairs of fixed points in T_1^2 ; the results for the cases when one or both pairs are not offset are obtained by setting $\tau_1^a = 0$ and/or $\tau_1^b = 0$.

6 Calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)$

6.1 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 1) \bmod 2$

In this case $(i_1^a, i_2^a), (i_1^b, i_2^b) = (45)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned} (45)(16) \circ (45)(16) &= (45)(45) \circ (45)(45) = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2[1 + (-1)^{\tau_2^a + \tau_2^b}][m_2^a n_2^b - n_2^a m_2^b + \\ &+ (-1)^{\tau_1^a + 1}(n_2^a n_2^b + m_2^a m_2^b + m_2^a n_2^b) + (-1)^{\tau_1^b}(n_2^a n_2^b + m_2^a m_2^b + n_2^a m_2^b) + \\ &+ (-1)^{\tau_1^a + \tau_1^b}(m_2^a n_2^b - n_2^a m_2^b)] \end{aligned} \quad (105)$$

$$(45)(16) \circ (45)(45) = 0 \quad (106)$$

6.2 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 0) \bmod 2$

In this case $(i_1^a, i_2^a), (i_1^b, i_2^b) = (56)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (14)$ or (56) .

$$\begin{aligned} (56)(14) \circ (56)(14) &= (56)(56) \circ (56)(56) = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2[1 + (-1)^{\tau_2^a + \tau_2^b}][m_2^a n_2^b - n_2^a m_2^b][1 + (-1)^{\tau_1^a + \tau_1^b}] + \\ &+ (-1)^{\tau_1^a + 1}(n_2^a n_2^b + m_2^a m_2^b + m_2^a n_2^b) + (-1)^{\tau_1^b}(n_2^a n_2^b + m_2^a m_2^b + n_2^a m_2^b) \end{aligned} \quad (107)$$

$$(56)(14) \circ (56)(56) = 0 \quad (108)$$

6.3 $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0) \bmod 2, (n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$

In this case $(i_1^a, i_2^a) = (56)$, $(j_1^a, j_2^a) = (14)$ or (56) , and $(i_1^b, i_2^b) = (45)$, $(j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned} (56)(14) \circ (45)(16) &= (-1)^{\tau_2^a} (56)(14) \circ (45)(45) = \\ &= (-1)^{\tau_2^b} (56)(56) \circ (45)(45) = (-1)^{\tau_2^a + \tau_2^b} (56)(56) \circ (45)(16) = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2 \left[-(n_2^a n_2^b + m_2^a m_2^b + m_2^a n_2^b)[1 + (-1)^{\tau_1^a + \tau_1^b}] + \right. \\ &\left. + (-1)^{\tau_1^a}(n_2^a n_2^b + m_2^a m_2^b + n_2^a m_2^b) + (-1)^{\tau_1^b + 1}(n_2^a m_2^b - m_2^a n_2^b) \right] \end{aligned} \quad (109)$$

7 Computations

Using the calculations presented in the previous section and the appendices, we may compute the intersection numbers $a \circ b$ and $a \circ b'$ for any two stacks a and b . We seek wrapping numbers $(n_k^{a,b}, m_k^{a,b})$ ($k = 1, 2, 3$) that determine the bulk parts $\Pi_{a,b}^{\text{bulk}}$ and exceptional parts $\Pi_{a,b}^{\text{ex}}$ of the two supersymmetric stacks, that have no symmetric matter and not too much antisymmetric matter on either stack, *i.e.* they satisfy the constraints (94),(95),(96) and (97), and that produce the required intersection numbers

$$(|a \circ b|, |a \circ b'|) = (1, 2) \quad (110)$$

On all lattices, it turns out that this is only possible when the wrapping numbers of the two stacks are in *different* classes mod 2, *i.e.* only the $(n_1^a, m_1^a) = (1, 0) \bmod 2 = (n_3^a, m_3^a)$, $(n_1^b, m_1^b) = (1, 1) \bmod 2 = (n_3^b, m_3^b)$ sector can satisfy the constraints. It follows that the fixed points associated with Π_a^{ex} and Π_b^{ex} are

$$(i_1^a, i_2^a)(j_1^a, j_2^a) = (14)(14), (14)(56), (56)(14), \text{ or } (56)(56) \quad (111)$$

$$(i_1^b, i_2^b)(j_1^b, j_2^b) = (16)(16), (16)(45), (45)(16), \text{ or } (45)(45) \quad (112)$$

As noted previously, it suffices to consider only the offset pairs of fixed points $(i_1^a, i_2^a) = (56)$ and $(i_1^b, i_2^b) = (45)$ in T_1^2 , since the results for the non-offset pairs may be obtained by setting $\tau_1^a = 0$ and/or $\tau_1^b = 0$. Further, we need only perform the computations for the offset pairs of fixed points $(j_1^a, j_2^a) = (56)$ and $(j_1^b, j_2^b) = (45)$ in T_3^2 , since the results for $(a \circ b, a \circ b')$ in the other sectors are merely special cases that arise when $\tau_2^a = 0$ and/or $\tau_2^b = 0$. If, for example, $\tau_2^a = 1 \bmod 2$ is needed in order to satisfy one or more of the constraints, then we *must* use the offset fixed points $(j_1^a, j_2^a) = (56)$; otherwise, we may use either (14) or (56). Similarly, if $\tau_2^b = 1 \bmod 2$ then $(j_1^b, j_2^b) = (45)$ must be used; otherwise (16) or (45) may be used. Thus, in all cases the exceptional parts of the stacks are taken to be

$$\Pi_a^{\text{ex}} = (56)(56)(n_2^a, m_2^a) = (-1)^{\tau_0^a} \{ [-(n_2^a + m_2^a) + (-1)^{\tau_1^a} n_2^a] [\epsilon_5 + (-1)^{\tau_2^a} \epsilon_6] - [n_2^a + (-1)^{\tau_1^a} m_2^a] [\tilde{\epsilon}_5 + (-1)^{\tau_2^a} \tilde{\epsilon}_6] \} \quad (113)$$

$$\Pi_b^{\text{ex}} = (45)(45)(n_2^b, m_2^b) = (-1)^{\tau_0^b} \{ [m_2^b + (-1)^{\tau_1^b+1} (n_2^b + m_2^b)] [\epsilon_4 + (-1)^{\tau_2^b} \epsilon_5] + [n_2^b + m_2^b + (-1)^{\tau_1^b+1} n_2^b] [\tilde{\epsilon}_4 + (-1)^{\tau_2^b} \tilde{\epsilon}_5] \} \quad (114)$$

The orientifold duals $\Pi_{a,b}^{\text{ex} \prime}$ of course depend upon the lattice.

7.1 AAB lattice

For this lattice

$$\Pi_a^{\text{ex} \prime} = (56)(56)(n_2^a, m_2^a)' = (-1)^{\tau_0^a} \{ [(n_2^a + m_2^a) - (-1)^{\tau_1^a} n_2^a] [\epsilon_6 + (-1)^{\tau_2^a} \epsilon_5] + [m_2^a - (-1)^{\tau_1^a} (n_2^a + m_2^a)] [\tilde{\epsilon}_6 + (-1)^{\tau_2^a} \tilde{\epsilon}_5] \} \quad (115)$$

$$\Pi_b^{\text{ex} \prime} = (45)(45)(n_2^b, m_2^b)' = (-1)^{\tau_0^b} \{ [-m_2^b + (-1)^{\tau_1^b} (n_2^b + m_2^b)] [\epsilon_4 + (-1)^{\tau_2^b} \epsilon_6] + [n_2^b + (-1)^{\tau_1^b} m_2^b] [\tilde{\epsilon}_4 + (-1)^{\tau_2^b} \tilde{\epsilon}_6] \} \quad (116)$$

On all lattices there are many sets of wrapping numbers for which the constraints (94) and (96) that ensure the absence of symmetric matter on both stacks are satisfied. However, the vast majority of these do not have the required intersection numbers (110). Here, and generally on the other lattices, the constraint (97) that limits the amount of matter in antisymmetric representations of the gauge group, eliminates a lot of otherwise acceptable solutions. Consider, for example,

$$(n_1^a, m_1^a)(n_2^a, m_2^a)(n_3^a, m_3^a) = (1, 0)(1, 0)(1, 0) \quad (117)$$

$$(n_1^b, m_1^b)(n_2^b, m_2^b)(n_3^b, m_3^b) = (1, 1)(1, 1)(1, -1) \quad (118)$$

so that

$$\Pi_a^{\text{bulk}} = \rho_1 \quad (119)$$

$$\Pi_b^{\text{bulk}} = 3(\rho_1 + \rho_3 - \rho_4 - \rho_6) \quad (120)$$

On the **AAB** lattice these give

$$\Pi_a^{\text{bulk}'} = \rho_1 \quad (121)$$

$$\Pi_b^{\text{bulk}'} = -3\rho_6 \quad (122)$$

Combining with the exceptional parts given in (113),(114),(115) and (116), we have

$$a = \frac{1}{2}\rho_1 + \frac{1}{2}(-1)^{\tau_0^a} \{[-1 + (-1)^{\tau_1^a}][\epsilon_5 + (-1)^{\tau_2^a}\epsilon_6] - [\tilde{\epsilon}_5 + (-1)^{\tau_2^a}\tilde{\epsilon}_6]\} \quad (123)$$

$$a' = \frac{1}{2}\rho_1 + \frac{1}{2}(-1)^{\tau_0^a + \tau_1^a + \tau_2^a} \{[-1 + (-1)^{\tau_1^a}][\epsilon_5 + (-1)^{\tau_2^a}\epsilon_6] - [\tilde{\epsilon}_5 + (-1)^{\tau_2^a}\tilde{\epsilon}_6]\} \quad (124)$$

$$b = \frac{3}{2}(\rho_1 + \rho_3 - \rho_4 - \rho_6) + \frac{1}{2}(-1)^{\tau_0^b} \{[1 + (-1)^{\tau_1^b+1}2][\epsilon_4 + (-1)^{\tau_2^b}\epsilon_5] + [2 + (-1)^{\tau_1^b+1}][\tilde{\epsilon}_4 + (-1)^{\tau_2^b}\tilde{\epsilon}_5]\} \quad (125)$$

$$b' = -\frac{3}{2}\rho_6 + \frac{1}{2}(-1)^{\tau_0^b} \{[-1 + (-1)^{\tau_1^b}2][\epsilon_4 + (-1)^{\tau_2^b}\epsilon_6] + [1 + (-1)^{\tau_1^b}][\tilde{\epsilon}_4 + (-1)^{\tau_2^b}\tilde{\epsilon}_6]\} \quad (126)$$

Then both (94) and, when $\tau_1^b = 0$, (96) are satisfied, so that there is no matter in the symmetric representation of either gauge group. In fact, we find that $g(A_p^a)$, defined in (92), is zero, so that $a \circ \Pi_{O6} = 0$. As noted earlier, this means that there is no matter in the antisymmetric representation of the gauge group on the stack a either. It is obvious from (117) that a preserves supersymmetry, because the angles ϕ_k^a ($k = 1, 2, 3$) that the 1-cycles with wrapping numbers (n_k^a, m_k^a) on T_k^2 make with the $\text{Re } z_k$ axis are all zero, independently of the complex structure U_3 . However, $\text{Im } U_3$ is fixed when the supersymmetry constraint $Y^b = 0$, with Y^b given in Table 8, is imposed on b . Since, from (51), the real part $\text{Re } U_3 = \frac{1}{2}$ we find

$$U_3 = \frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\pi/3} \quad (127)$$

Also $X^b > 0$ so that b is supersymmetric, as required. In fact, from (118), $\phi_1^b = \phi_2^b = \pi/6$, $\phi_3^b = -\pi/3$. Thus T_3^2 has the same complex structure as $T_{1,2}^2$, given in (47), and is therefore an $SU(3)$ lattice too. Finally, it is easy to verify from (101) and (102) using (109) and (147) that

$$a \circ b = \frac{3}{2} + \frac{1}{2}(-1)^{\tau_0^a + \tau_0^b + \tau_2^b} [2 - (-1)^{\tau_1^a}] \quad (128)$$

$$a \circ b' = -\frac{3}{2} + \frac{1}{2}(-1)^{\tau_0^a + \tau_0^b + \tau_2^a + \tau_2^b} [1 - 2(-1)^{\tau_1^a}] \quad (129)$$

Consequently $(a \circ b, a \circ b') = (2, -1)$ or $(1, -2)$, provided that $\tau_1^a = 0$ and $\tau_2^a = 1 \pmod{2}$. (It is often, but not invariably, the case that we find two pairs of values of $(a \circ b, a \circ b')$ both of which satisfy (110).) However, in this example $g(A_p^b) = 12$, so that (97) is *not* satisfied, and there are too many copies of the antisymmetric representation \mathbf{A}_b to obtain the standard model matter content without vector-like quarks or leptons.

In fact, on this lattice we find just two independent pairs of wrapping number sets which, with a suitable choice of Wilson lines, satisfy all of the constraints. They are shown in Table 14. The first pair yields $(a \circ b, a \circ b') = (2, 1)$ or $(-1, -2)$, and the second pair $(-2, 1)$ or $(-1, 2)$. For both pairs there are sets of wrapping numbers that differ from those displayed by sign reversal of the pairs (n_k^a, m_k^a) on two of the tori T_k^2 ($k = 1, 2, 3$) and similarly for (n_k^b, m_k^b) , and which also satisfy all of the constraints with a possibly different choice of Wilson lines. This is the case in all of our displayed solutions. We display just one representative in each set. In all cases, $a \circ b$ and $a \circ b'$ is the same for all representatives.

As in the example discussed above, there is no matter in the antisymmetric representation \mathbf{A}_a on stack a in either of our solutions. The two solutions have the same wrapping numbers on b for which the number of antisymmetric representations is

$$\#(\mathbf{A}_b) = \frac{1}{2}g(A_p^b) = 2 \quad (130)$$

$(n_1^a, m_1^a; n_2^a, m_2^a; n_3^a, m_3^a)$	$(A_1^a, A_3^a, A_4^a, A_6^a)$	$(n_1^b, m_1^b; n_2^b, m_2^b; n_3^b, m_3^b)$	$(A_1^b, A_3^b, A_4^b, A_6^b)$	$\text{Im } U_3$
$(1, 0; 1, 0; 1, 0)$	$(1, 0, 0, 0)$	$(1, -1; 1, -1; -1, 1)$	$(1, 1, -1, -1)$	$\frac{\sqrt{3}}{2}$
$(1, -2; 1, 0; -1, 2)$	$(1, 2, -2, -4)$	$(1, -1; 1, -1; -1, 1)$	$(1, 1, -1, -1)$	$\frac{\sqrt{3}}{2}$

Table 14: Solutions on the **AAB** lattice.

$(n_1^a, m_1^a; n_2^a, m_2^a; n_3^a, m_3^a)$	$(A_1^a, A_3^a, A_4^a, A_6^a)$	$(n_1^b, m_1^b; n_2^b, m_2^b; n_3^b, m_3^b)$	$(A_1^b, A_3^b, A_4^b, A_6^b)$	$\text{Im } U_3$
$(1, 0; 1, -1; -1, 2)$	$(0, 1, 0, -2)$	$(1, -1; 1, -1; -1, 1)$	$(1, 1, -1, -1)$	$\frac{1}{2\sqrt{3}}$
$(1, -2; 1, -1; -1, 0)$	$(2, 1, 0, 0)$	$(1, -1; 1, -1; -1, 1)$	$(1, 1, -1, -1)$	$\frac{1}{2\sqrt{3}}$

Table 15: Solutions on the **BAB** lattice.

When $a \circ b$ and $a \circ b'$ have the same sign, as in the first pair, $N_b = 2$ and $\mathbf{A}_b = \bar{\mathbf{1}}$. Thus, in the first case there are no quark singlet states q_L^c on a , only the $U(3)$ gauge particles; and there are 2 left-chiral lepton singlet states ℓ_L^c on the stack b , besides the $U(2)$ gauge particles. In contrast, in the second set where $a \circ b$ and $a \circ b'$ have opposite signs, $N_b = 3$ and $\mathbf{A}_b = \bar{\mathbf{3}}$. Thus there are no lepton singlet states ℓ_L^c on a , only the $U(2)$ gauge particles; and there are 2 left-chiral quark singlet states q_L^c on the stack b , besides the $U(3)$ gauge particles. As before, the supersymmetry constraint requires that the complex structure $U_3 = e^{i\pi/3}$, so that again T_3^2 is an $SU(3)$ lattice. As noted in the introduction, the antisymmetric representation carries $Q = 2$ units of the relevant $U(1)$ charge. In the latter case, then, the 2 left-chiral quark singlet states q_L^c contribute $Q_b = 4$ units of $U(1)_b$ charge. However, the 3 quark doublets contribute $Q_b = 6$ and the remaining 4 left-chiral quark singlets, arising from intersections of the stack b with stacks c, d, \dots each having just one D6-brane, contribute $Q_b = -4$. Thus the overall cancellation of the charge Q_b (required by RR tadpole cancellation) can *not* be achieved in this case. A similar argument applies in the former case when 2 left-chiral lepton singlet states arise in the antisymmetric representation on the $U(2)$ stack.

7.2 BAB lattice

For this lattice

$$\begin{aligned} \Pi_a^{\text{ex}'} = (56)(56)(n_2^a, m_2^a)' &= (-1)^{\tau_0^a} \{ [-m_2^a + (-1)^{\tau_1^a} (n_2^a + m_2^a)] [\epsilon_6 + (-1)^{\tau_2^a} \epsilon_5] + \\ &\quad + [n_2^a + (-1)^{\tau_1^a} m_2^a] [\tilde{\epsilon}_6 + (-1)^{\tau_2^a} \tilde{\epsilon}_5] \} \end{aligned} \quad (131)$$

$$\begin{aligned} \Pi_b^{\text{ex}'} = (45)(45)(n_2^b, m_2^b)' &= (-1)^{\tau_0^b} \{ [-n_2^b + (-1)^{\tau_1^b+1} m_2^b] [\epsilon_4 + (-1)^{\tau_2^b} \epsilon_6] + \\ &\quad - [n_2^b + m_2^b + (-1)^{\tau_1^b} n_2^b] [\tilde{\epsilon}_4 + (-1)^{\tau_2^b} \tilde{\epsilon}_6] \} \end{aligned} \quad (132)$$

Here too we find two independent pairs of wrapping number sets which, with a suitable choice of Wilson lines, satisfy all of the constraints. They are shown in Table 15. Both pairs have the property that there is no matter in the antisymmetric representation \mathbf{A}_a on stack a , and that there are $\#(\mathbf{A}_b) = 2$ on stack b , as in (130). Also, $a \circ b$ and $a \circ b'$ have opposite signs for both pairs. Thus in both solutions there are 2 quark singlet states q_L^c on b , besides the $U(3)$ gauge particles. As for the **AAB** lattice, overall cancellation of Q_b can *not* be achieved in this case either. The imaginary part $\text{Im } U_3$ of the complex structure on T_3^2 is fixed, as before, by the stack b . In neither case is an $SU(3)$ lattice required.

7.3 ABB lattice

As noted previously, for this lattice the orientifold duals $\Pi_{a,b}^{\text{ex}'}$ of the exceptional parts differ only by an overall sign from those on the **BAB** lattice. Thus we merely reverse the signs of the right hand sides of (131) and (132). Nevertheless different solutions do occur, as is apparent from the three solutions displayed in Table 16. In the first place, $(n_2^b, m_2^b) = (0, 1) \bmod 2$ on this lattice, rather than $(1, 1) \bmod 2$ on the **BAB** lattice. This means that the exceptional part of b is guaranteed to be different on the two

$(n_1^a, m_1^a; n_2^a, m_2^a; n_3^a, m_3^a)$	$(A_1^a, A_3^a, A_4^a, A_6^a)$	$(n_1^b, m_1^b; n_2^b, m_2^b; n_3^b, m_3^b)$	$(A_1^b, A_3^b, A_4^b, A_6^b)$	$\text{Im } U_3$
$(1, 0; 1, 1; 1, 0)$	$(2, 1, 0, 0)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{1}{2\sqrt{3}}$
$(1, -2; 1, 1; 1, -2)$	$(0, -3, 0, 6)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{1}{2\sqrt{3}}$
$(1, -2; 1, -1; -1, 0)$	$(2, 1, 0, 0)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{1}{2\sqrt{3}}$

Table 16: Solutions on the **ABB** lattice.

$(n_1^a, m_1^a; n_2^a, m_2^a; n_3^a, m_3^a)$	$(A_1^a, A_3^a, A_4^a, A_6^a)$	$(n_1^b, m_1^b; n_2^b, m_2^b; n_3^b, m_3^b)$	$(A_1^b, A_3^b, A_4^b, A_6^b)$	$\text{Im } U_3$
$(1, 0; 2, -1; 1, -2)$	$(1, -1, -2, 2)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{\sqrt{3}}{2}$
$(1, -2; 2, -1; -1, 0)$	$(3, 3, 0, 0)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{\sqrt{3}}{2}$
$(1, -2; 0, 1; 1, -2)$	$(1, -1, -2, 2)$	$(1, -1; 0, 1; 1, -1)$	$(1, 0, -1, 0)$	$-\frac{\sqrt{3}}{2}$

Table 17: Solutions on the **BBB** lattice.

lattices. In any case, since the function $g(A_p^a)$ is different on the two lattices, the constraints that ensure the absence of antisymmetric matter differ, and we should expect different bulk solutions for b , as indeed we find.

All three solutions share the property that, as before, there is no matter in the antisymmetric representation \mathbf{A}_a on stack a . However, unlike all of the previous solutions, all of the solutions in Table 16 also have the property that there is no matter in the antisymmetric representation \mathbf{A}_b on stack b . This might have been foreseen. The factor 12 that appears in the function $g(A_p^a)$ for the **ABB** lattice in Table 11 requires that the only solutions that satisfy the constraints (95) and (97) must indeed have $g(A_p^a) = 0 = g(A_p^b)$, and hence have no antisymmetric (or symmetric) matter on a or b . A similar argument can be applied to the **AAB** and **BAB** lattices in both of which the function $g(A_p^a)$ has a factor 4. Then satisfying the constraints (95) and (97) requires that $\#(\mathbf{A}_{a,b}) \leq 2$, as we found. In all of the solutions displayed in Table 16, $a \circ b$ and $a \circ b'$ have opposite sign, so that $N_b = 3$ and the gauge particles of $U(3)$ live on b . The complex structure U_3 of T_3^2 is fixed by the supersymmetry constraint on b , and, as we found for the **BAB** lattice, an $SU(3)$ lattice is not required. The first and third pair of solutions have the same bulk parts for both a and b . Nevertheless, they are distinct solutions since the exceptional parts of a differ in the two pairs.

7.4 BBB lattice

For this lattice

$$\begin{aligned} \Pi_a^{\text{ex}'} = (56)(56)(n_2^a, m_2^a)' &= (-1)^{\tau_0^a} \{ [n_2^a + (-1)^{\tau_1^a} m_2^a] [\epsilon_6 + (-1)^{\tau_2^a} \epsilon_5] + \\ &\quad + [(n_2^a + m_2^a) - (-1)^{\tau_1^a} n_2^a] [\tilde{\epsilon}_6 + (-1)^{\tau_2^a} \tilde{\epsilon}_5] \} \end{aligned} \quad (133)$$

$$\begin{aligned} \Pi_b^{\text{ex}'} = (45)(45)(n_2^b, m_2^b)' &= (-1)^{\tau_0^b} \{ [-(n_2^b + m_2^b) + (-1)^{\tau_1^b+1} n_2^b] [\epsilon_4 + (-1)^{\tau_2^b} \epsilon_6] + \\ &\quad - [m_2^b + (-1)^{\tau_1^b+1} (n_2^b + m_2^b)] [\tilde{\epsilon}_4 + (-1)^{\tau_2^b} \tilde{\epsilon}_6] \} \end{aligned} \quad (134)$$

Since $(n_2^a, m_2^a) = (0, 1) \bmod 2$ on this lattice only, the exceptional parts of any solutions are guaranteed to differ from all previous solutions. In this case again we find three solutions. They are displayed in Table 17. The factor of 12 that appears in the function $g(A_p^a)$ in Table 11 for this lattice too again ensures that any solution is guaranteed to have no antisymmetric (or symmetric) matter on either stack. Again $a \circ b$ and $a \circ b'$ have opposite sign in all solutions, so that $N_b = 3$ and the gauge particles of $U(3)$ live on b . The supersymmetry constraint requires that the complex structure $U_3 = e^{-i\pi/3}$, so that T_3^2 is an $SU(3)$ lattice in this case. As for the **ABB** lattice, the first and third pair of solutions are distinct since the exceptional parts of a differ.

8 Conclusions

We have shown that, unlike the \mathbb{Z}_6 orientifold, the \mathbb{Z}'_6 orientifold *can* support supersymmetric stacks a and b of D6-branes with intersection numbers satisfying $(|a \circ b|, |a \circ b'|) = (2, 1)$ or $(1, 2)$. Stacks having this property are an indispensable ingredient in any intersecting brane model that has *just* the matter content of the (supersymmetric) standard model. The number of branes, $N_{a,b}$ in stacks a, b is required to be $(N_a, N_b) = (3, 2)$ or $(2, 3)$ so as to produce the gauge groups $U(3)$ and $U(2)$ from which the QCD $SU(3)_c$ and the electroweak $SU(2)_L$ gauge fields emerge. By construction, in all of our solutions there is no matter in symmetric representations of the gauge groups on either stack. However, some of the solutions *do* have matter, quark singlets q_L^c or lepton singlets ℓ_L^c , in the antisymmetric representation of gauge group on one of the stacks. This is not possible on the \mathbb{Z}_6 orientifold because all supersymmetric D6-branes wrap the same bulk 3-cycle as the O6-planes, from which it follows that $a \circ \Pi_{O6} = 0$. Then, requiring the absence of symmetric matter necessarily entails the absence of antisymmetric matter too. In contrast, on the \mathbb{Z}'_6 orientifold there exist supersymmetric 3-cycles that do not wrap the O6-planes. Thus, there is more latitude in this case, and the solutions with antisymmetric matter exploit this feature. Unfortunately, however, none of the solutions of this nature that we have found can be enlarged to give just the standard-model spectrum, since the overall cancellation of the relevant $U(1)$ charge cannot be achieved with just this matter content. Nevertheless, some of our solutions have no antisymmetric (or symmetric) matter on either stack. We shall attempt in a future work to construct a realistic (supersymmetric) standard model using one of our solutions.

The presence of singlet matter on the branes in some, but not all, of our solutions is an important feature of our results. It is clear that different orbifold point groups produce different physics, as indeed, for the reasons just given, our results also illustrate. The point group must act as an automorphism of the lattice used, but it is less clear that realising a given point group symmetry on different lattices produces different physics. Our results indicate that different lattices may produce different physics, since, for example, the solutions with no antisymmetric (or symmetric) matter on either stack occur only on the **ABB** and **BBB** lattices, and we understand why any acceptable solutions without symmetric matter must also lack antisymmetric matter. The observation that the lattice does affect the physics suggests that other lattices are worth investigating in both the \mathbb{Z}_6 and \mathbb{Z}'_6 orientifolds. In particular, since Z_6 can be realised on a G_2 lattice, as well as on an $SU(3)$ lattice, one or more of all three $SU(3)$ lattices in the \mathbb{Z}_6 case, and of the two on $T_{1,2}^2$ in the \mathbb{Z}'_6 case, could be replaced by a G_2 lattice. We shall explore this avenue too in future work.

The construction of a realistic model will, of course, entail adding further stacks of D6-branes c, d, \dots , with just a single brane in each stack, arranging that the matter content is just that of the supersymmetric standard model, the whole set satisfying (one of) the conditions (55)...(62) and the corresponding condition in (65)...(72) for RR tadpole cancellation. In a supersymmetric orientifold RR tadpole cancellation ensures that NSNS tadpoles are also cancelled, but some moduli, (some of) of the complex structure moduli, the Kähler moduli and the dilaton, remain unstabilised. Recent developments have shown how such moduli may be stabilised using RR, NSNS and metric fluxes [22, 23, 24, 25, 26], and indeed Cámara, Font & Ibáñez [27, 28] have shown how models similar to the ones we have been discussing can be uplifted into ones with stabilised Kähler moduli using a “rigid corset”. In general, such fluxes contribute to tadpole cancellation conditions and might make them easier to satisfy. In contrast, the rigid corset can be added to any RR tadpole-free assembly of D6-branes in order to stabilise all moduli. Thus our results represent an important first step to obtaining a supersymmetric standard model from intersecting branes with all moduli stabilised.

9 Acknowledgments

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A Calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ on the AAB lattice

A.1 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 1) \bmod 2$

As in §6.1, the fixed points involved are $(i_1^a, i_2^a), (i_1^b, i_2^b) = (56)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (14)$ or (56)

$$\begin{aligned} (45)(16) \circ (45)(16)' &= (45)(45) \circ (45)(45)' = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2 [m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b + (-1)^{\tau_1^a + 1} (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) + \\ &+ (-1)^{\tau_1^b + 1} (n_2^a n_2^b + m_2^a n_2^b + n_2^a m_2^b) + (-1)^{\tau_1^a + \tau_1^b} (n_2^a n_2^b - m_2^a m_2^b)] \end{aligned} \quad (135)$$

$$\begin{aligned} (45)(16) \circ (45)(45)' &= (45)(45) \circ (45)(16)' = \\ &= (-1)^{\tau_0^a + \tau_0^b + \tau_2^a + \tau_2^b + 1} 2 \left[n_2^a m_2^b + m_2^a n_2^b + m_2^a m_2^b + \right. \\ &+ \left. [(-1)^{\tau_1^a + 1} + (-1)^{\tau_1^b + 1}] (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) (-1)^{\tau_1^a + \tau_1^b} (n_2^a n_2^b - m_2^a m_2^b) \right] \end{aligned} \quad (136)$$

A.2 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 0) \bmod 2$

In this case $(i_1^a, i_2^a), (i_1^b, i_2^b) = (56)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (14)$ or (56) , as in §6.2. Consider first the exceptional brane

$$\begin{aligned} (56)(14) &= (-1)^{\tau_0^a} \{[-(n_2^a + m_2^a) + (-1)^{\tau_1^a} n_2^a][\epsilon_1 + (-1)^{\tau_2^a} \epsilon_4] - \\ &- [n_2^a + (-1)^{\tau_1^a} m_2^a][\tilde{\epsilon}_1 + (-1)^{\tau_2^a} \tilde{\epsilon}_4]\} \end{aligned} \quad (137)$$

and its orientifold dual on the AAB lattice

$$\begin{aligned} (56)(14)' &= (-1)^{\tau_0^a} \{[n_2^a + m_2^a - (-1)^{\tau_1^a} n_2^a][\epsilon_1 + (-1)^{\tau_2^a} \epsilon_4] - \\ &- [m_2^a - (-1)^{\tau_1^a} (n_2^a + m_2^a)][\tilde{\epsilon}_1 + (-1)^{\tau_2^a} \tilde{\epsilon}_4]\} \end{aligned} \quad (138)$$

Since $(n_2^a, m_2^a) = (1, 0) \bmod 2$ on this lattice

$$(56)(14) = (\tilde{\epsilon}_1 + \tilde{\epsilon}_4) \bmod 2 \quad (139)$$

$$(56)(14)' = (\tilde{\epsilon}_1 + \tilde{\epsilon}_4) \bmod 2 \quad (140)$$

$$(141)$$

Likewise

$$(56)(56) = (\tilde{\epsilon}_5 + \tilde{\epsilon}_6) \bmod 2 \quad (142)$$

$$(56)(56)' = (\tilde{\epsilon}_5 + \tilde{\epsilon}_6) \bmod 2 \quad (143)$$

$$(144)$$

It follows that for all of the allowed exceptional branes in this sector

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b) = 0 \bmod 8 \quad (145)$$

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)' = 0 \bmod 8 \quad (146)$$

The first of these may be verified from the results in §6.2, using $(n_2^{a,b}, m_2^{a,b}) = (1, 0) \bmod 2$. Further, $A_1^{a,b} = 1 \bmod 2$ and $A_{3,4,6}^{a,b} = 0 \bmod 2$ in this sector. In fact, since $A_1^{a,b} A_6^{a,b} = A_3^{a,b} A_4^{a,b}$, it follows that $A_6^{a,b} = 0 \bmod 4$. Then $f_{AB} = 0 \bmod 8 = f_{AB'}$. Hence, from (101) and (102), we see that $a \circ b = 0 \bmod 2 = a \circ b'$ and we cannot obtain the required odd intersection number from this sector. We therefore omit the calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ in this sector.

A.3 $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0) \bmod 2$, $(n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$

As in §6.3, $(i_1^a, i_2^a) = (56)$, $(j_1^a, j_2^a) = (14)$ or (56) , and $(i_1^b, i_2^b) = (45)$, $(j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned}
(56)(14) &\circ (45)(16)' = (-1)^{\tau_2^a} (56)(14) \circ (45)(45)' = \\
&= (-1)^{\tau_2^a + \tau_2^b} (56)(56) \circ (45)(45)' = (-1)^{\tau_2^b} (56)(56) \circ (45)(16)' = \\
&= (-1)^{\tau_0^a + \tau_0^b + 1} 2 \left[-(n_2^a n_2^b + m_2^a n_2^b + n_2^a m_2^b) + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (n_2^a n_2^b - m_2^a m_2^b) + \right. \\
&\quad \left. + (-1)^{\tau_1^a + \tau_1^b} (m_2^a m_2^b + m_2^a n_2^b + n_2^a m_2^b) \right] \tag{147}
\end{aligned}$$

B Calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ on the BAB lattice

B.1 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 1) \bmod 2$

As in §6.1, $(i_1^a, i_2^a), (i_1^b, i_2^b) = (45)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned}
(45)(16) &\circ (45)(16)' = (45)(45) \circ (45)(45)' = \\
&= (-1)^{\tau_0^a + \tau_0^b} 2 \left[m_2^a m_2^b - n_2^a n_2^b + [(-1)^{\tau_1^a + 1} + (-1)^{\tau_1^b + 1}] (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) + \right. \\
&\quad \left. + (-1)^{\tau_1^a + \tau_1^b} (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) \right] \tag{148}
\end{aligned}$$

$$\begin{aligned}
(45)(16) &\circ (45)(45)' = (45)(45) \circ (45)(16)' = \\
&= (-1)^{\tau_0^a + \tau_0^b + \tau_2^a + \tau_2^b} 2 \left[m_2^a m_2^b - n_2^a n_2^b [(-1)^{\tau_1^a + 1} + (-1)^{\tau_1^b + 1}] (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) + \right. \\
&\quad \left. + (-1)^{\tau_1^a + \tau_1^b} (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) \right] \tag{149}
\end{aligned}$$

B.2 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 0) \bmod 2$

As in §6.2, the relevant fixed points are $(i_1^a, i_2^a), (i_1^b, i_2^b) = (56)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (14)$ or (56) . It follows from (137) that, since $(n_2^a, m_2^a) = (1, 1) \bmod 2$ on this lattice

$$(56)(14) = (\epsilon_1 + \epsilon_4) \bmod 2 \tag{150}$$

and, using Table 7, that

$$(56)(14)' = (\epsilon_1 + \epsilon_4) \bmod 2 \tag{151}$$

Likewise

$$(56)(56) = (\epsilon_5 + \epsilon_6) \bmod 2 \tag{152}$$

$$(56)(56)' = (\epsilon_5 + \epsilon_6) \bmod 2 \tag{153}$$

$$\tag{154}$$

Thus, as in §6.2, for all of the allowed exceptional branes in this sector

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b) = 0 \bmod 8 \tag{155}$$

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)' = 0 \bmod 8 \tag{156}$$

Again, the first of these may be verified from the results of §6.2 using $(n_2^{a,b}, m_2^{a,b}) = (1, 1) \bmod 2$. Further, $A_3^{a,b} = 1 \bmod 2$ and $A_{1,4,6}^{a,b} = 0 \bmod 2$ in this sector. Since $A_1^{a,b} A_6^{a,b} = A_3^{a,b} A_4^{a,b}$, it follows that $A_4^{a,b} = 0 \bmod 4$, and again $f_{AB} = 0 \bmod 8 = f_{AB'}$. Thus, from (101) and (102), we see that $a \circ b = 0 \bmod 2 = a \circ b'$ and, as before, we cannot obtain the required odd intersection number from this sector.

B.3 $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0)$, $(n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$

As in §6.3, the relevant fixed points are the fixed points involved are $(i_1^a, i_2^a) = (56)$, $(j_1^a, j_2^a) = (14)$ or (56) , and $(i_1^b, i_2^b) = (45)$, $(j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned}
(56)(14) &\circ (45)(16)' = (-1)^{\tau_2^a} (56)(14) \circ (45)(45)' = \\
&= (-1)^{\tau_2^a + \tau_2^b} (56)(56) \circ (45)(45)' = (-1)^{\tau_2^b} (56)(56) \circ (45)(16)' = \\
&= (-1)^{\tau_0^a + \tau_0^b + 1} 2 \left[m_2^a m_2^b + m_2^a n_2^b + n_2^a m_2^b \right] + (-1)^{\tau_1^a + \tau_1^b + 1} (m_2^a m_2^b - n_2^a n_2^b) + \\
&+ \left[(-1)^{\tau_1^a + 1} + (-1)^{\tau_1^b + 1} \right] (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) \quad (157)
\end{aligned}$$

C Calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ on the ABB lattice

The fixed points involved are the same as for the **BAB** (and the **AAB**) lattice. Although the change in lattice *does* affect the calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$, it does so only trivially. From Table 7, we see that the only difference between the **BAB** lattice and the **ABB** lattice is that there is an overall minus sign in the orientifold image of the exceptional branes. Thus, the results for this sector are trivially obtained by changing the overall sign for the calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ in §B. Of course, this does not imply that the value $a \circ b'$ is also reversed in sign, since the contribution $f_{AB'}$ from the bulk branes is the same for both lattices.

D Calculations of $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)'$ on the BBB lattice

D.1 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 1) \bmod 2$

As in §6.1, $(i_1^a, i_2^a), (i_1^b, i_2^b) = (45)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned}
(45)(16) &\circ (45)(16)' = (45)(45) \circ (45)(45)' = \\
&= (-1)^{\tau_0^a + \tau_0^b} 2 \left[n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (n_2^a n_2^b - m_2^a m_2^b) \right] + \\
&+ (-1)^{\tau_1^a + \tau_1^b} (m_2^a m_2^b + m_2^a n_2^b + n_2^a m_2^b) \quad (158)
\end{aligned}$$

$$\begin{aligned}
(45)(16) &\circ (45)(45)' = (45)(45) \circ (45)(16)' = \\
&= (-1)^{\tau_0^a + \tau_0^b + \tau_2^a + \tau_2^b} 2 \left[n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (n_2^a n_2^b - m_2^a m_2^b) \right] + \\
&+ (-1)^{\tau_1^a + \tau_1^b} (m_2^a m_2^b + m_2^a n_2^b + n_2^a m_2^b) \quad (159)
\end{aligned}$$

D.2 $(n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (1, 0) \bmod 2$

As in §6.2, the relevant fixed points are $(i_1^a, i_2^a), (i_1^b, i_2^b) = (56)$ and $(j_1^a, j_2^a), (j_1^b, j_2^b) = (14)$ or (56) . It follows from (137) and Table 7 that, since $(n_2^a, m_2^a) = (0, 1) \bmod 2$ on this lattice

$$(56)(14) = (\epsilon_1 + \epsilon_4 + \tilde{\epsilon}_1 + \tilde{\epsilon}_4) \bmod 2 \quad (160)$$

$$(56)(14)' = (\epsilon_1 + \epsilon_4 + \tilde{\epsilon}_1 + \tilde{\epsilon}_4) \bmod 2 \quad (161)$$

$$(162)$$

Likewise

$$(56)(56) = (\epsilon_5 + \epsilon_6 + \tilde{\epsilon}_5 + \tilde{\epsilon}_6) \bmod 2 \quad (163)$$

$$(56)(56)' = (\epsilon_5 + \epsilon_6 + \tilde{\epsilon}_5 + \tilde{\epsilon}_6) \bmod 2 \quad (164)$$

$$(165)$$

Thus, as in §A.2, for all of the allowed exceptional branes in this sector

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b) = 0 \bmod 8 \quad (166)$$

$$(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b)(j_1^b, j_2^b)' = 0 \bmod 8 \quad (167)$$

However, $A_{1,3}^{a,b} = 1 \bmod 2$ and $A_{4,6}^{a,b} = 0 \bmod 2$ in this sector, from which we can *not* conclude that either f_{AB} or $f_{AB'}$ is 0 mod 8. In this case, therefore, we must compute $(i_1^a, i_2^a)(j_1^a, j_2^a) \circ (i_1^b, i_2^b), (j_1^b, j_2^b)'$.

$$\begin{aligned} (56)(14) \circ (56)(14)' &= (-1)^{\tau_2^b} (56)(56) \circ (56)(56)' = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2[1 + (-1)^{\tau_2^a + \tau_2^b}] \left[-(m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) + \right. \\ &\quad \left. + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}](n_2^a n_2^b + m_2^a n_2^b + n_2^a m_2^b) + (-1)^{\tau_1^a + \tau_1^b} (m_2^a m_2^b - n_2^a n_2^b) \right] \end{aligned} \quad (168)$$

$$(56)(14) \circ (56)(56)' = 0 = (56)(56) \circ (56)(14)' \quad (169)$$

D.3 $(n_1^a, m_1^a) = (n_3^a, m_3^a) = (1, 0), (n_1^b, m_1^b) = (n_3^b, m_3^b) = (1, 1) \bmod 2$

As in § 6.3, the relevant fixed points are the fixed points involved are $(i_1^a, i_2^a) = (56), (j_1^a, j_2^a) = (14)$ or (56) , and $(i_1^b, i_2^b) = (45), (j_1^b, j_2^b) = (16)$ or (45) .

$$\begin{aligned} (56)(14) \circ (45)(16)' &= (-1)^{\tau_2^a} (56)(14) \circ (45)(45)' = \\ &= (-1)^{\tau_2^a + \tau_2^b} (56)(56) \circ (45)(45)' = (-1)^{\tau_2^b} (56)(56) \circ (45)(16)' = \\ &= (-1)^{\tau_0^a + \tau_0^b + 1} 2 \left[n_2^a n_2^b - m_2^a m_2^b + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}](m_2^a m_2^b + m_2^a n_2^b + n_2^a m_2^b) + \right. \\ &\quad \left. + (-1)^{\tau_1^a + \tau_1^b + 1} (n_2^a n_2^b + m_2^a n_2^b + n_2^a m_2^b) \right] \end{aligned} \quad (170)$$

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